61A Lecture 9

Wednesday, February 11

Announcements

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- Guerrilla section this Saturday 2/14 on recursion (Please RSVP on Piazza!)

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- Midterm 1 solutions are posted; grades will be released soon

Data Abstraction

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Data Abstraction

All Programmers

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All Programmers

Great Programmers

Rational Numbers

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Rational Numbers

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Exact representation of fractions

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As soon as division occurs, the exact representation may be lost! (Demo)

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Assume we can compose and decompose rational numbers:

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`

Rational Numbers

$$\frac{\text{numerator}}{\text{denominator}}$$

Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`

Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Assume we can compose and decompose rational numbers:

Constructor → `rational(n, d)` returns a rational number x

- `numer(x)` returns the numerator of x
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Rational Numbers

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor

- `rational(n, d)` returns a rational number x

Selectors

- `numer(x)` returns the numerator of x
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Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy}$$

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Rational Number Arithmetic

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Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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General Form

Rational Number Arithmetic Implementation

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

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- `rational(n, d)` returns a rational number x
- `numer(x)` returns the numerator of x
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Rational Number Arithmetic Implementation

```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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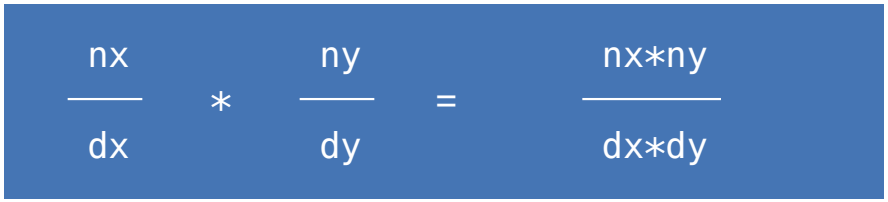

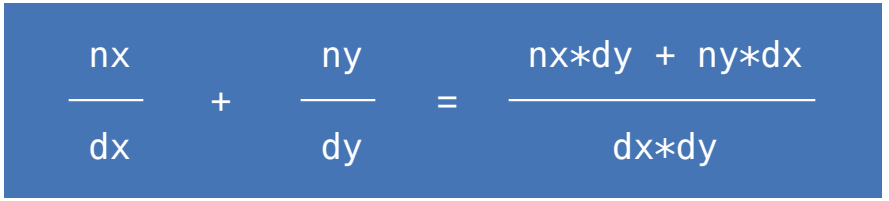

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These functions implement an abstract data type for rational numbers

Rational Number Arithmetic Implementation

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def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```

Constructor

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def print_rational(x):
    print(numer(x), '/', denom(x))
```

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def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

Constructor

Selectors

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Pairs

Representing Pairs Using Lists

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```
>>> pair = [1, 2]
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A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
```

A list literal:
Comma-separated expressions in brackets

"Unpacking" a list

Representing Pairs Using Lists

```
>>> pair = [1, 2]
>>> pair
[1, 2]

>>> x, y = pair
>>> x
1
>>> y
2

>>> pair[0]
1
>>> pair[1]
2
```

A list literal:
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"Unpacking" a list

Representing Pairs Using Lists

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>>> pair = [1, 2]
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[1, 2]
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A list literal:
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>>> x
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>>> y
2
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"Unpacking" a list

```
>>> pair[0]
1
>>> pair[1]
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```

Element selection using the selection operator

Representing Pairs Using Lists

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>>> pair = [1, 2]
>>> pair
[1, 2]
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A list literal:
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>>> x, y = pair
>>> x
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>>> y
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```

"Unpacking" a list

```
>>> pair[0]
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>>> pair[1]
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```

Element selection using the selection operator

```
>>> from operator import getitem
```

Representing Pairs Using Lists

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>>> pair = [1, 2]
>>> pair
[1, 2]
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A list literal:
Comma-separated expressions in brackets

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>>> x, y = pair
>>> x
1
>>> y
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```

"Unpacking" a list

```
>>> pair[0]
1
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Element selection using the selection operator

```
>>> from operator import getitem
>>> getitem(pair, 0)
1
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```
>>> pair = [1, 2]
>>> pair
[1, 2]
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A list literal:
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>>> x, y = pair
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>>> pair[0]
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Element selection function

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Element selection function

More lists next lecture

Representing Rational Numbers

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def rational(n, d):
    """Construct a rational number that represents N/D."""
    return [n, d]
```

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Construct a list

Representing Rational Numbers

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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
```

Representing Rational Numbers

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def rational(n, d):
    """Construct a rational number that represents N/D."""
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Construct a list

```
def numer(x):
    """Return the numerator of rational number X."""
    return x[0]

def denom(x):
    """Return the denominator of rational number X."""
    return x[1]
```

Representing Rational Numbers

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def rational(n, d):
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```

Select item from a list

Representing Rational Numbers

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Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

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$$\frac{3}{2} * \frac{5}{3}$$

Reducing to Lowest Terms

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$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2}$$

$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2}$$

Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10}$$

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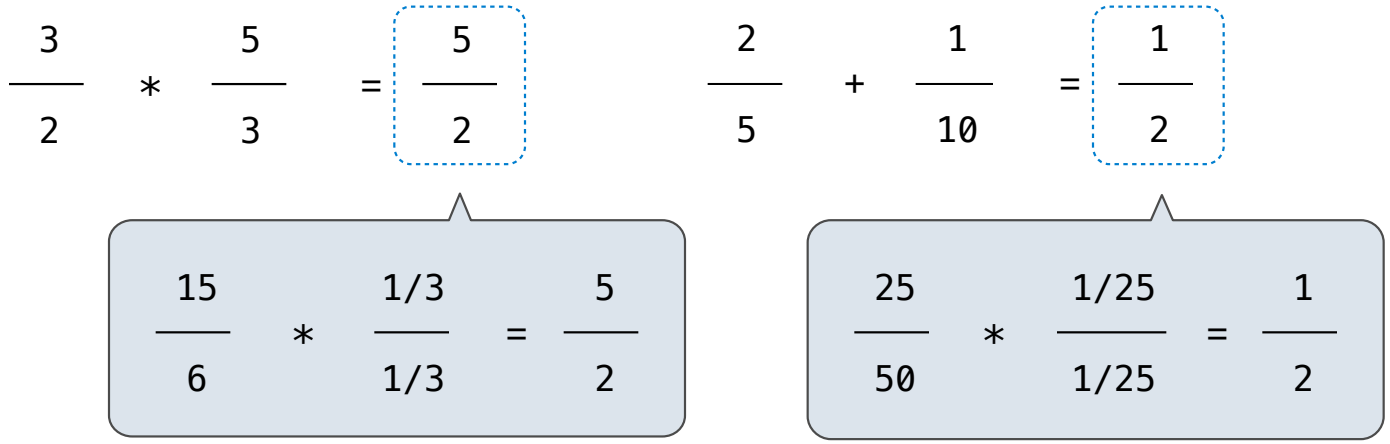
```
from fractions import gcd
```

Reducing to Lowest Terms

Example:

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from fractions import gcd

def rational(n, d):
```

Reducing to Lowest Terms

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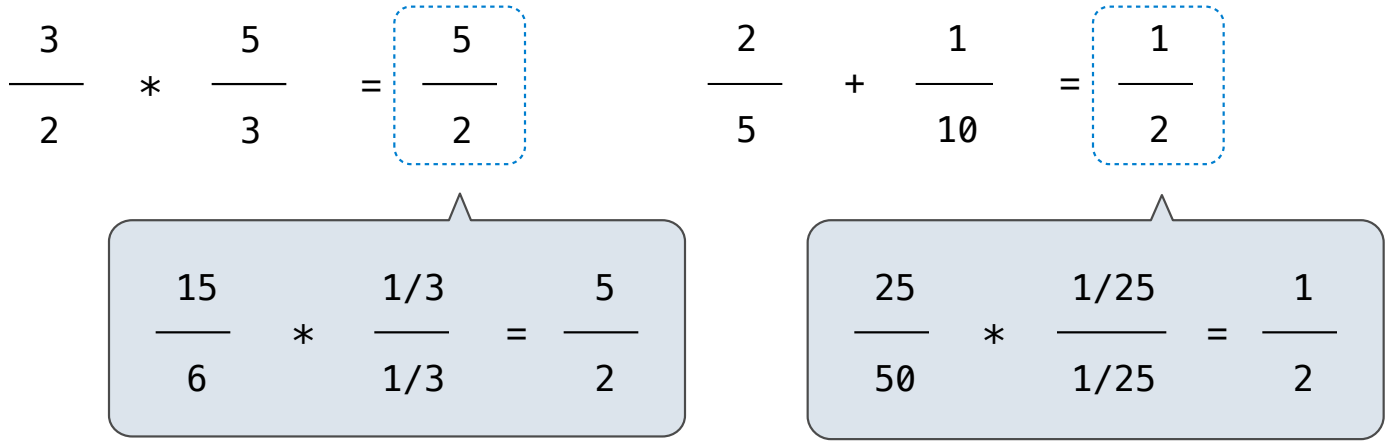


```
from fractions import gcd

def rational(n, d):
    """Construct a rational number x that represents n/d."""
```

Reducing to Lowest Terms

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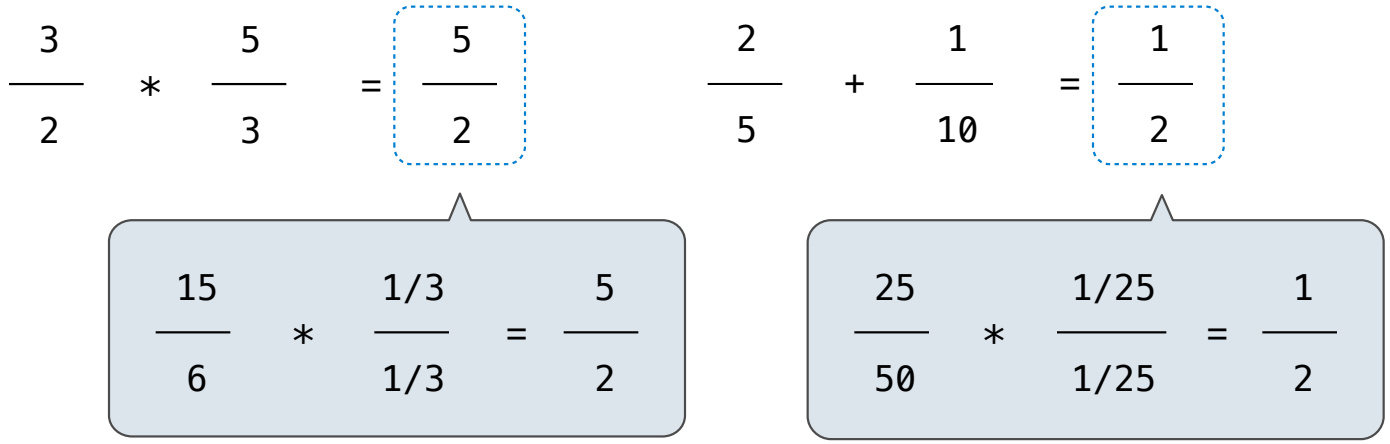


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from fractions import gcd

def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
```

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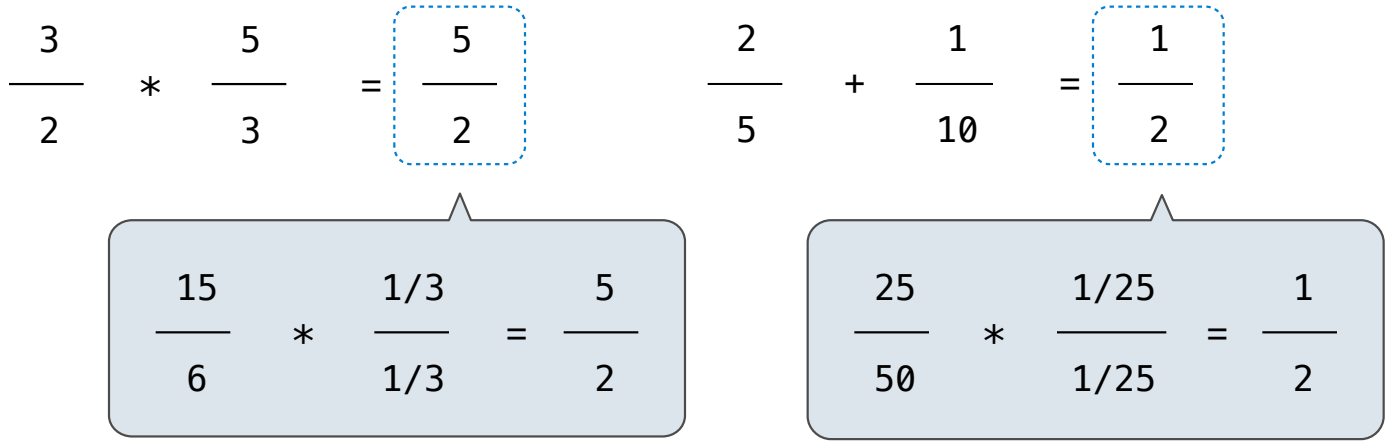


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from fractions import gcd

def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return [n//g, d//g]
```

Reducing to Lowest Terms

Example:

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$$\frac{15}{6} * \frac{1/3}{1/3} = \frac{5}{2} \qquad \frac{25}{50} * \frac{1/25}{1/25} = \frac{1}{2}$$

```
from fractions import gcd
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Greatest common divisor

```
def rational(n, d):
    """Construct a rational number x that represents n/d."""
    g = gcd(n, d)
    return [n//g, d//g]
```

Abstraction Barriers

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Parts of the program that...	Treat rationals as...	Using...

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Use rational numbers to perform computation		

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Implementation of lists

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Implementation of lists

Violating Abstraction Barriers

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add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
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Violating Abstraction Barriers

Does not use constructors

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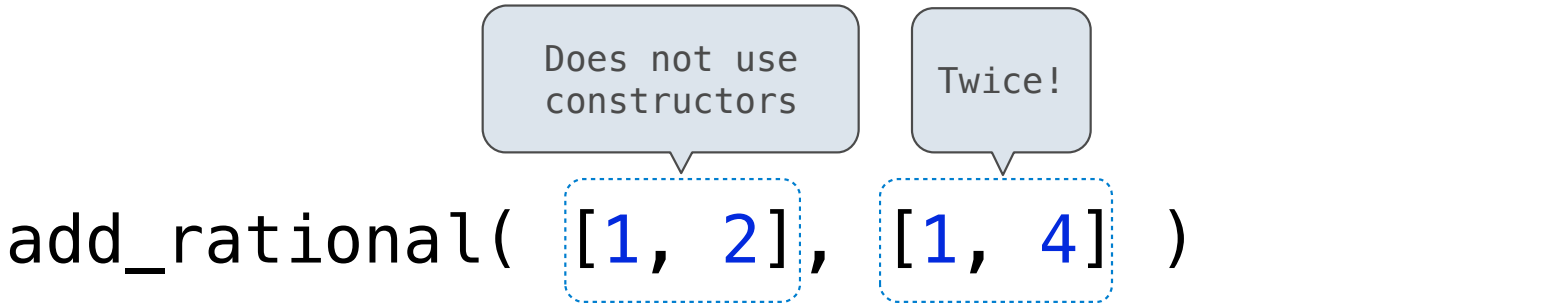
Violating Abstraction Barriers

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Violating Abstraction Barriers

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Twice!

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Violating Abstraction Barriers

Data Representations

What is Data?

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(Demo)

Rational Data Abstraction Implemented as Functions

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def rational(n, d):
    def select(name):
        if name == 'n':
            return n
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            return d
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def numer(x):
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numer(x)
```

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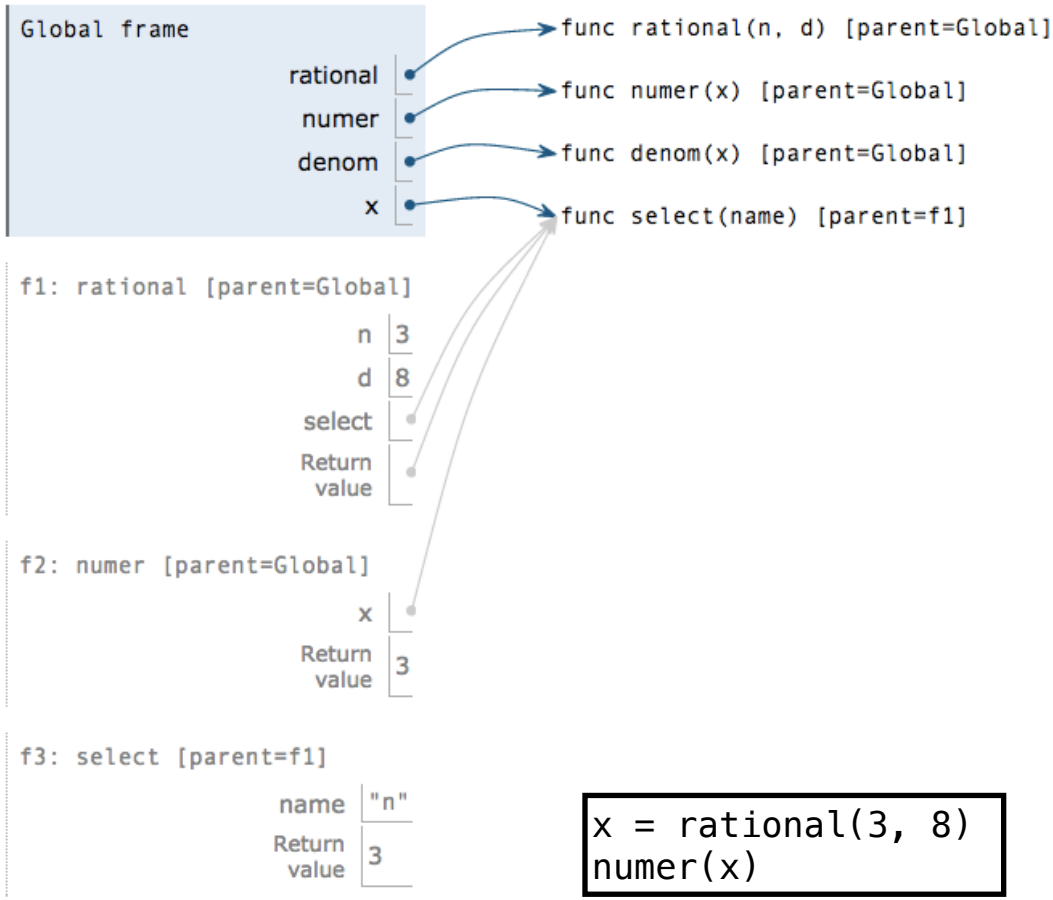


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Interactive Diagram