

## 61A Lecture 17

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Wednesday, March 4

## Announcements

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- Midterm 2 is on Thursday 3/19 7pm–9pm

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  - Emphasis: mutable data, object-oriented programming, recursion, and recursive data



## Generic Functions of Multiple Arguments

## More Generic Functions

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A function might want to operate on multiple data types

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- Type dispatching
- Type coercion

*What's different?* Today's generic functions apply to multiple arguments that don't share a common interface.

## Rational Numbers

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```
class Rational:
    """A rational number represented as a numerator and denominator."""
    def __init__(self, numer, denom):
        g = gcd(numer, denom)
        self.numer = numer // g
        self.denom = denom // g


    def __repr__(self):
        return 'Rational({0}, {1})'.format(self.numer, self.denom)
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    def add(self, other):
        nx, dx = self.numer, self.denom
        ny, dy = other.numer, other.denom
        return Rational(nx * dy + ny * dx, dx * dy)

    def mul(self, other):
        numer = self.numer * other.numer
        denom = self.denom * other.denom
        return Rational(numer, denom)
```

Greatest common  
divisor

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$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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(Demo)

# Complex Numbers

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## Complex Numbers

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class Complex:
    def add(self, other):
        return ComplexRI(self.real + other.real,
                          self.imag + other.imag)
    def mul(self, other):
        return ComplexMA(self.magnitude * other.magnitude,
                          self.angle + other.angle)
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```
class ComplexRI(Complex):
    """A rectangular representation."""
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag

    @property
    def magnitude(self):
        return (self.real ** 2 + self.imag ** 2) ** 0.5

    @property
    def angle(self):
        return atan2(self.imag, self.real)
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    @property
    def angle(self):
        return atan2(self.imag, self.real)
```

```
class ComplexMA(Complex):
    """A polar representation."""
    def __init__(self, magnitude, angle):
        self.magnitude = magnitude
        self.angle = angle

    @property
    def real(self):
        return self.magnitude * cos(self.angle)

    @property
    def imag(self):
        return self.magnitude * sin(self.angle)
```



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class Complex:
    def add(self, other):
        return ComplexRI(self.real + other.real,
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(Demo)

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>>> Rational(3, 14).add(Rational(2, 7))  
Rational(1, 2)
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```
>>> ComplexRI(0, 1).mul(ComplexMA(1, 0.5 * pi))  
ComplexMA(1, 1 * pi)
```

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>>> Rational(3, 14).add(Rational(2, 7))  
Rational(1, 2)  $\frac{3}{14} + \frac{2}{7}$   
  
>>> ComplexRI(0, 1).mul(ComplexMA(1, 0.5 * pi))  
ComplexMA(1, 1 * pi)  $i \cdot i$ 
```

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	<code>Rational(1, 2)</code>	
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	<code>&gt;&gt;&gt; Rational(1, 2) + ComplexRI(0.5, 2)</code>	$\frac{1}{2} + (0.5 + 2 \cdot i)$
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Operators	<code>&gt;&gt;&gt; Rational(1, 2) + ComplexRI(0.5, 2)</code>	$\frac{1}{2} + (0.5 + 2 \cdot i)$
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Operators	<code>&gt;&gt;&gt; ComplexMA(2, 0.5 * pi) * Rational(3, 2)</code>	
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Operators	<code>&gt;&gt;&gt; ComplexMA(2, 0.5 * pi) * Rational(3, 2)</code>	$2 \cdot i \cdot \frac{3}{2}$
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Operators	<pre>&gt;&gt;&gt; Rational(3, 14) + Rational(2, 7) Rational(1, 2)</pre>	$\frac{3}{14} + \frac{2}{7}$
	<pre>&gt;&gt;&gt; ComplexRI(0, 1) * ComplexMA(1, 0.5 * pi) ComplexMA(1, 1 * pi)</pre>	$i \cdot i$
Cross-type arithmetic	<pre>&gt;&gt;&gt; Rational(1, 2) + ComplexRI(0.5, 2) ComplexRI(1, 2)</pre>	$\frac{1}{2} + (0.5 + 2 \cdot i)$
	<pre>&gt;&gt;&gt; ComplexMA(2, 0.5 * pi) * Rational(3, 2) ComplexMA(3, 0.5 * pi)</pre>	$2 \cdot i \cdot \frac{3}{2}$

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```
>>> zero, one, two = 0, 1, 2
```

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>>> one + two
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Certain names are special because they have built-in behavior

These names always start and end with two underscores

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## Special Method Names in Python

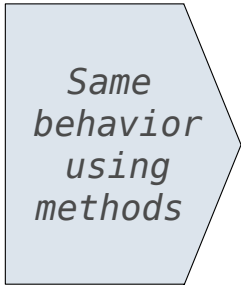
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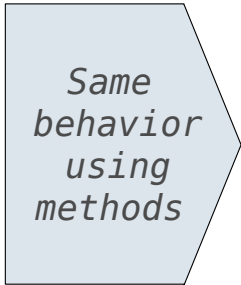
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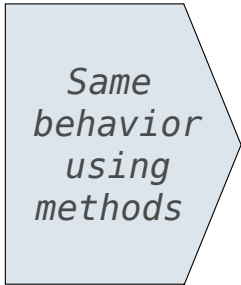
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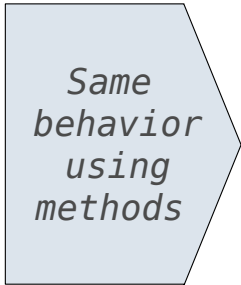
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*Same  
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```
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>>> one.__add__(two)
3
>>> zero.__bool__(), one.__bool__()
(False, True)
```

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Adding instances of user-defined classes invokes the `__add__` method

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class Number:
    """A number."""
    def __add__(self, other):
        return self.add(other)

    def __mul__(self, other):
        return self.mul(other)
```

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class Number:
    """A number."""
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```

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class Rational(Number):
    def add(self, other):
        ...
    def mul(self, other):
        ...
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>>> Rational(1, 3) + Rational(1, 6)
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## Special Methods

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Rational(1, 2)
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class Rational(Number):
    def add(self, other):
        ...
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        ...
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```
class Complex(Number):
    def add(self, other):
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We can also `__add__` complex numbers, even with multiple representations (Demo)

## Special Methods

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Adding instances of user-defined classes invokes the `__add__` method

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We can also `__add__` complex numbers, even with multiple representations (Demo)

<http://getpython3.com/diveintopython3/special-method-names.html>

<http://docs.python.org/py3k/reference/datamodel.html#special-method-names>

## Type Dispatching

## The Independence of Data Types

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Data abstraction and class definitions keep types separate

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*How do we add a complex number and a rational number together?*

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```
def add_complex_and_rational(c, r):  
    """Return c + r for complex c and rational r."""  
    return ComplexRI(c.real + r.numer/r.denom, c.imag)
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Define a different function for each possible combination of types for which an operation (e.g., addition) is valid

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class Number:  
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            return self.cross_apply(other, self.adders)
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    def cross_apply(self, other, cross_fns):  
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(Demo)

## Type Dispatching Analysis



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Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

## Type Coercion

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Idea: Some types can be converted into other types

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Question: Can any two numeric types be coerced into a common type?

Question: Is coercion exact?

## Applying Operators with Coercion

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```
class Number:
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## Applying Operators with Coercion

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```
class Number:  
    def __add__(self, other):  
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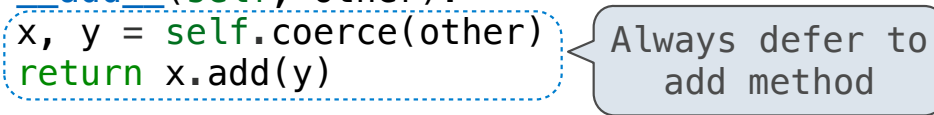
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class Number:  
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Same interface: no change required



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            return self, other
        elif (self.type_tag, other.type_tag) in self.coercions:
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        elif (self.type_tag, other.type_tag) in self.coercions:
            return (self.coerce_to(other.type_tag), other)
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## Applying Operators with Coercion

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            return (self.coerce_to(other.type_tag), other)

    def coerce_to(self, other_tag):
        coercion_fn = self.coercions[(self.type_tag, other_tag)]
        return coercion_fn(self)

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<b>Arg 1</b>	<b>Arg 2</b>	<b>Add</b>	<b>Multiply</b>
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		

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Arg 1	Arg 2	Add	Multiply
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Rational	Rational		
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From	To	Coerce
Complex	Rational	
Rational	Complex	

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Arg 1	Arg 2	Add	Multiply
Complex	Complex		
Rational	Rational		
Complex	Rational		
Rational	Complex		



From	To	Coerce
Complex	Rational	
Rational	Complex	

Type	Add	Multiply
Complex		
Rational		