61A Lecture 19

Monday, March 9

Announcements

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  - Fill out conflict form if you cannot attend due to a course conflict

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

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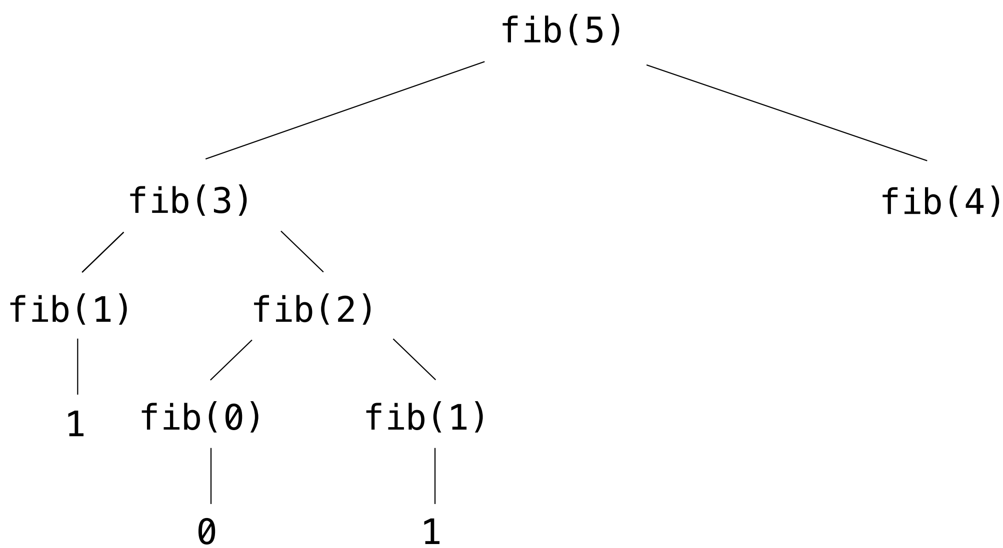


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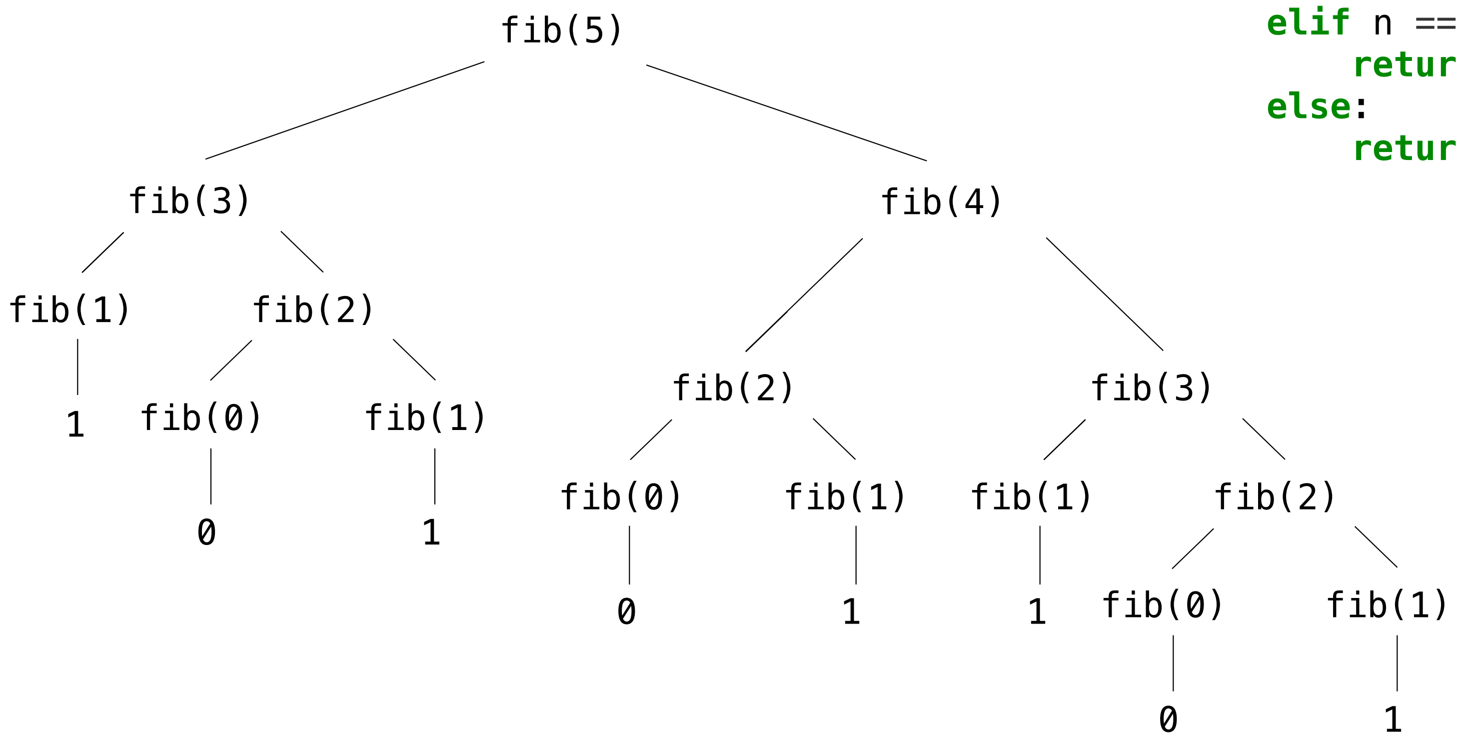

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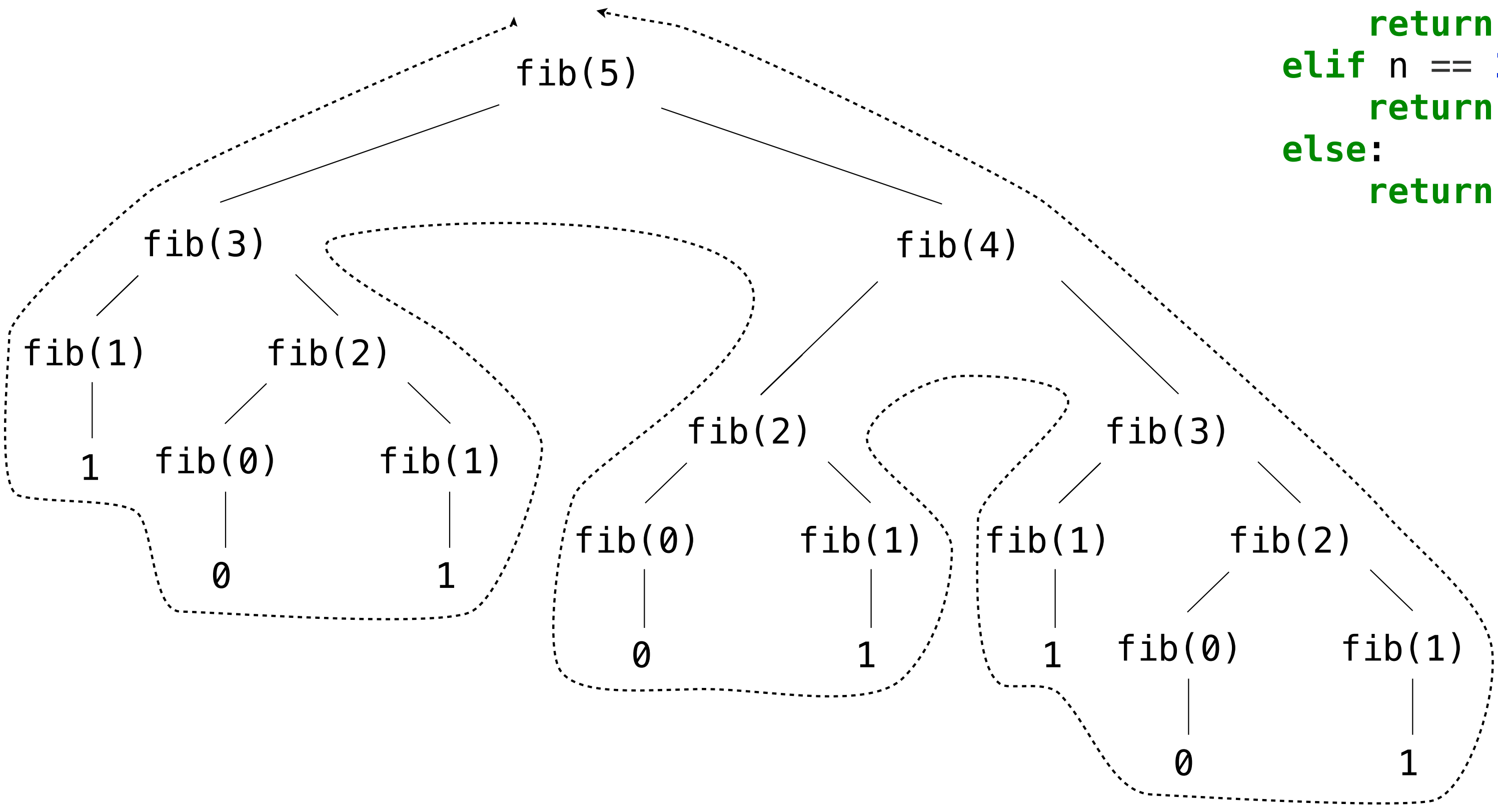

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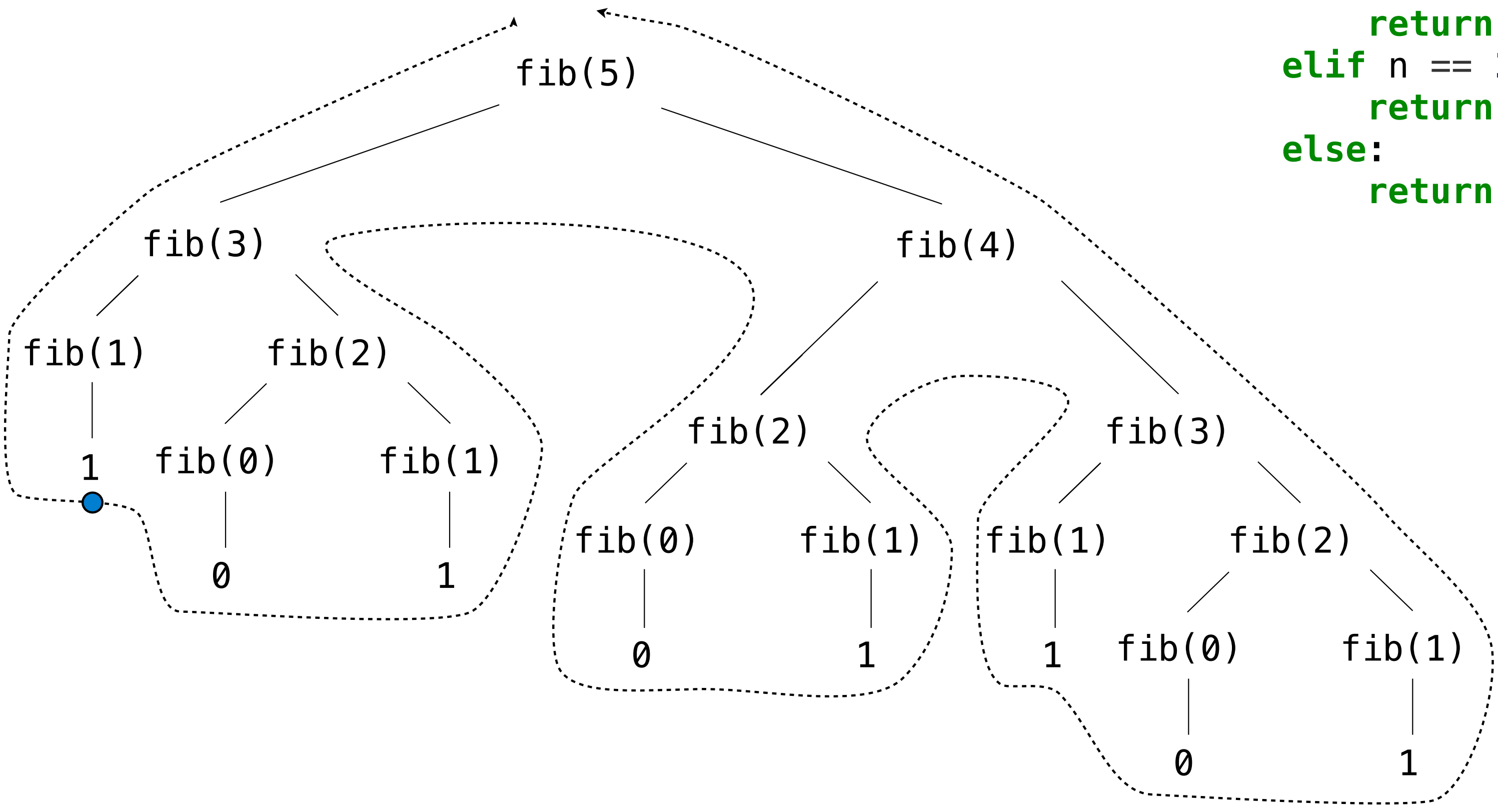

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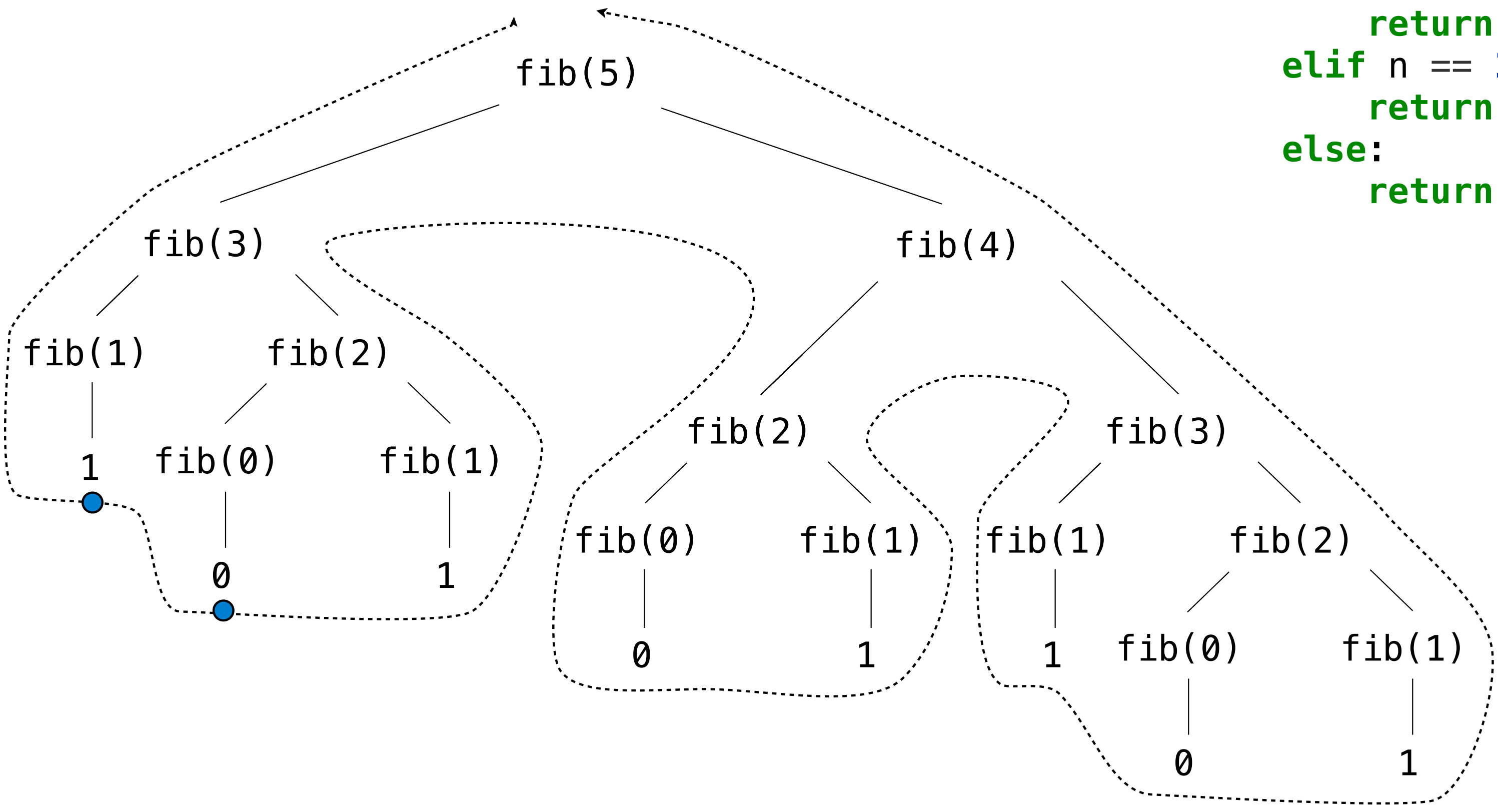

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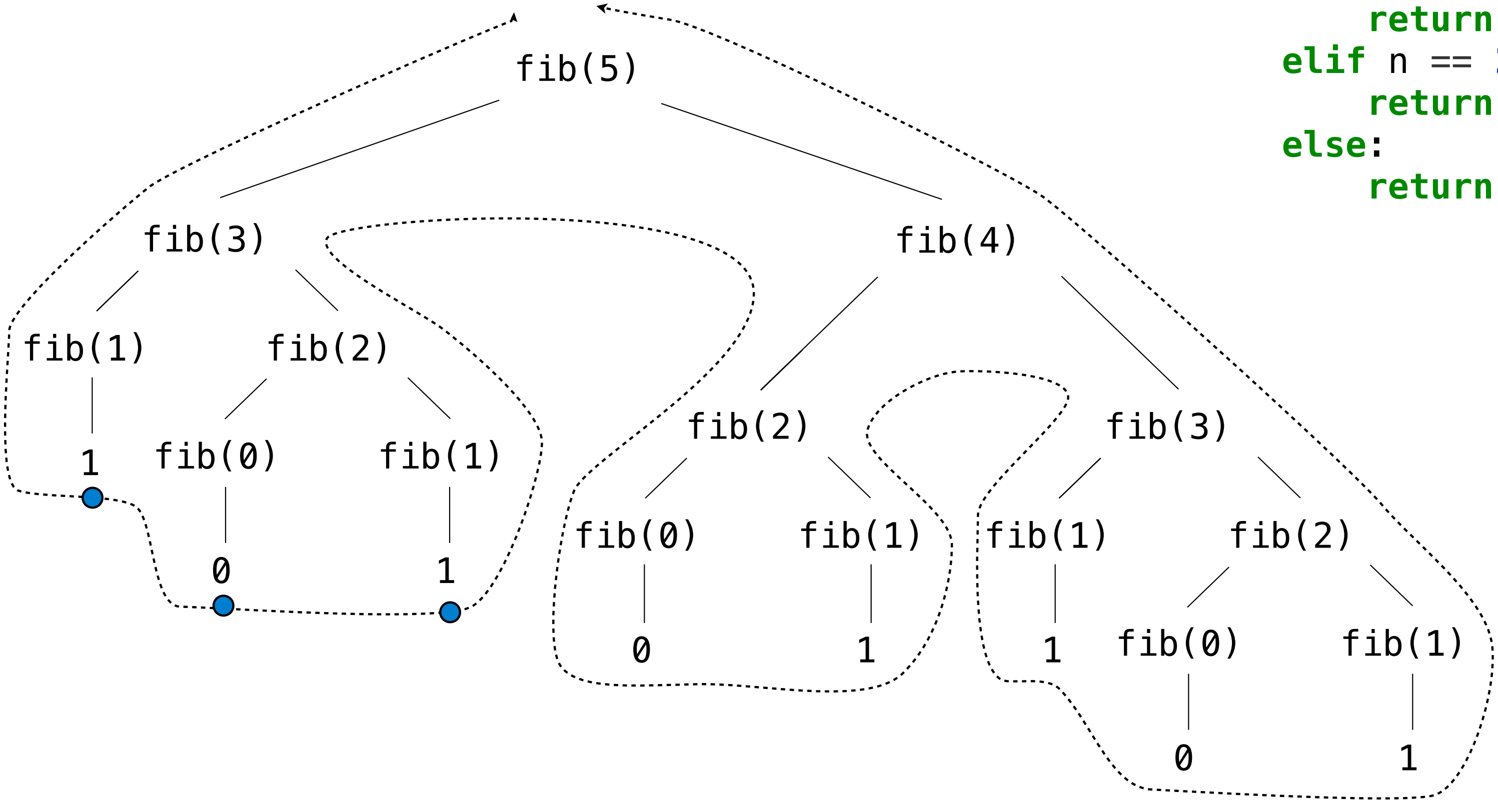

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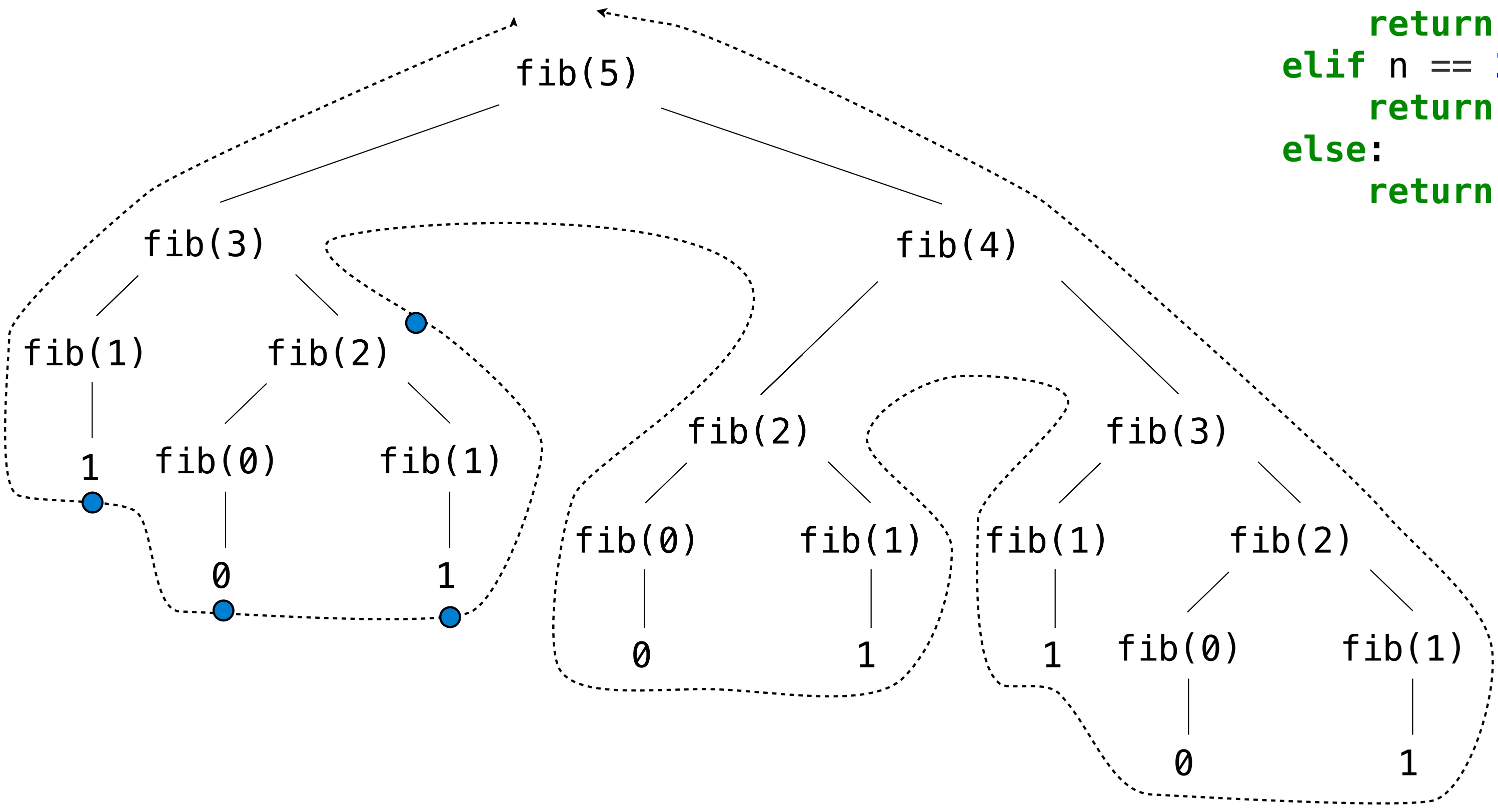

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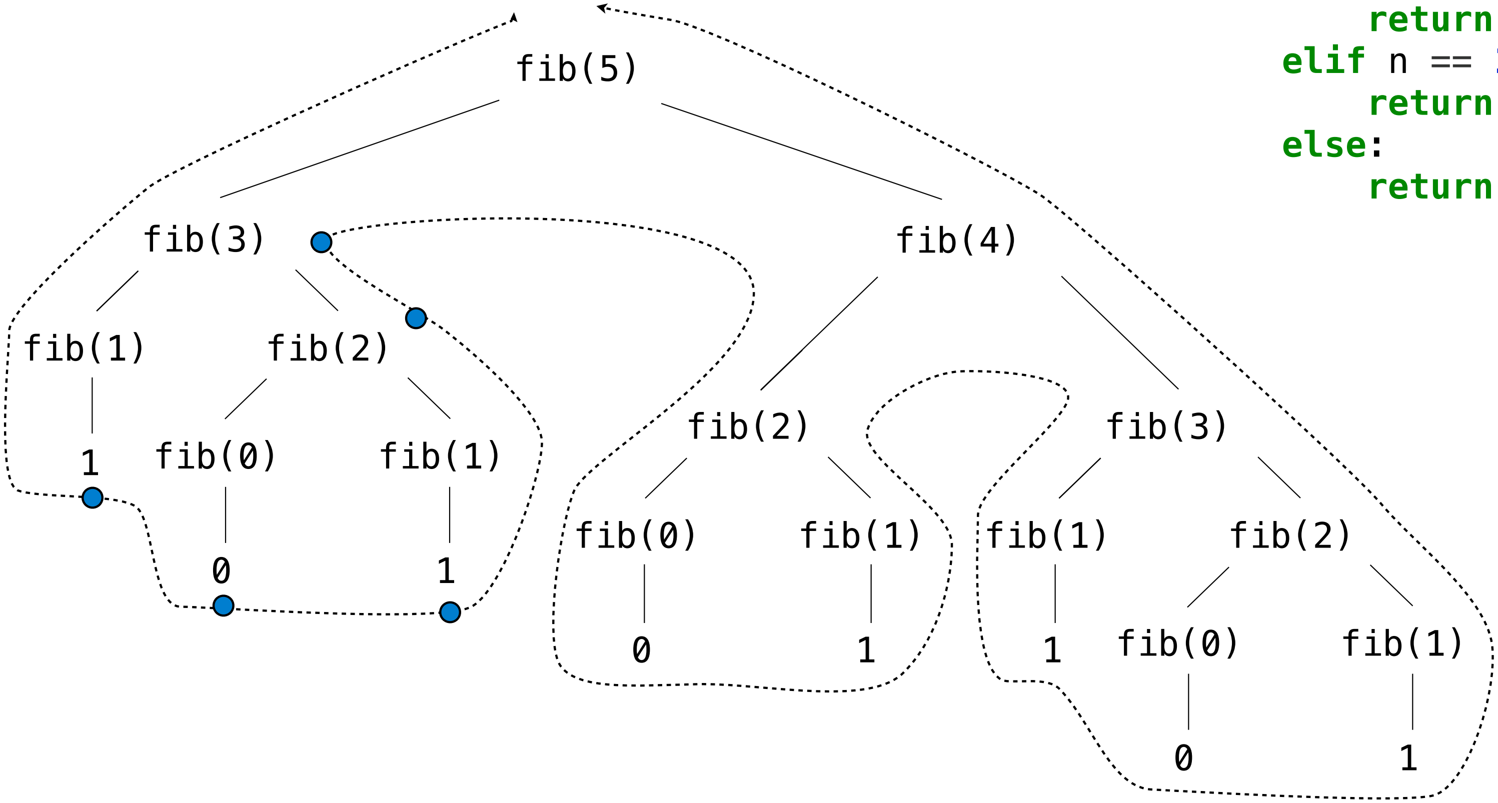


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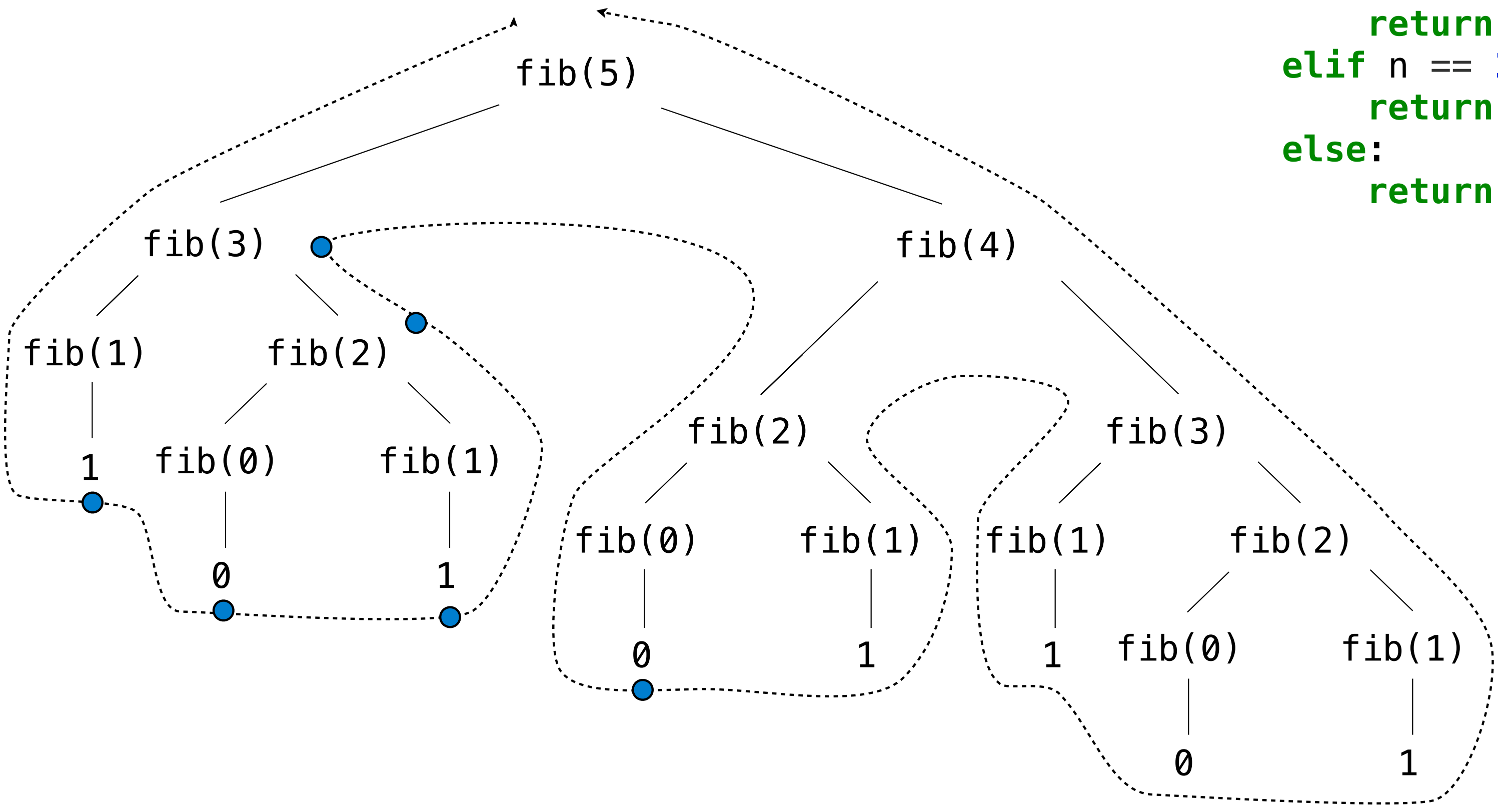

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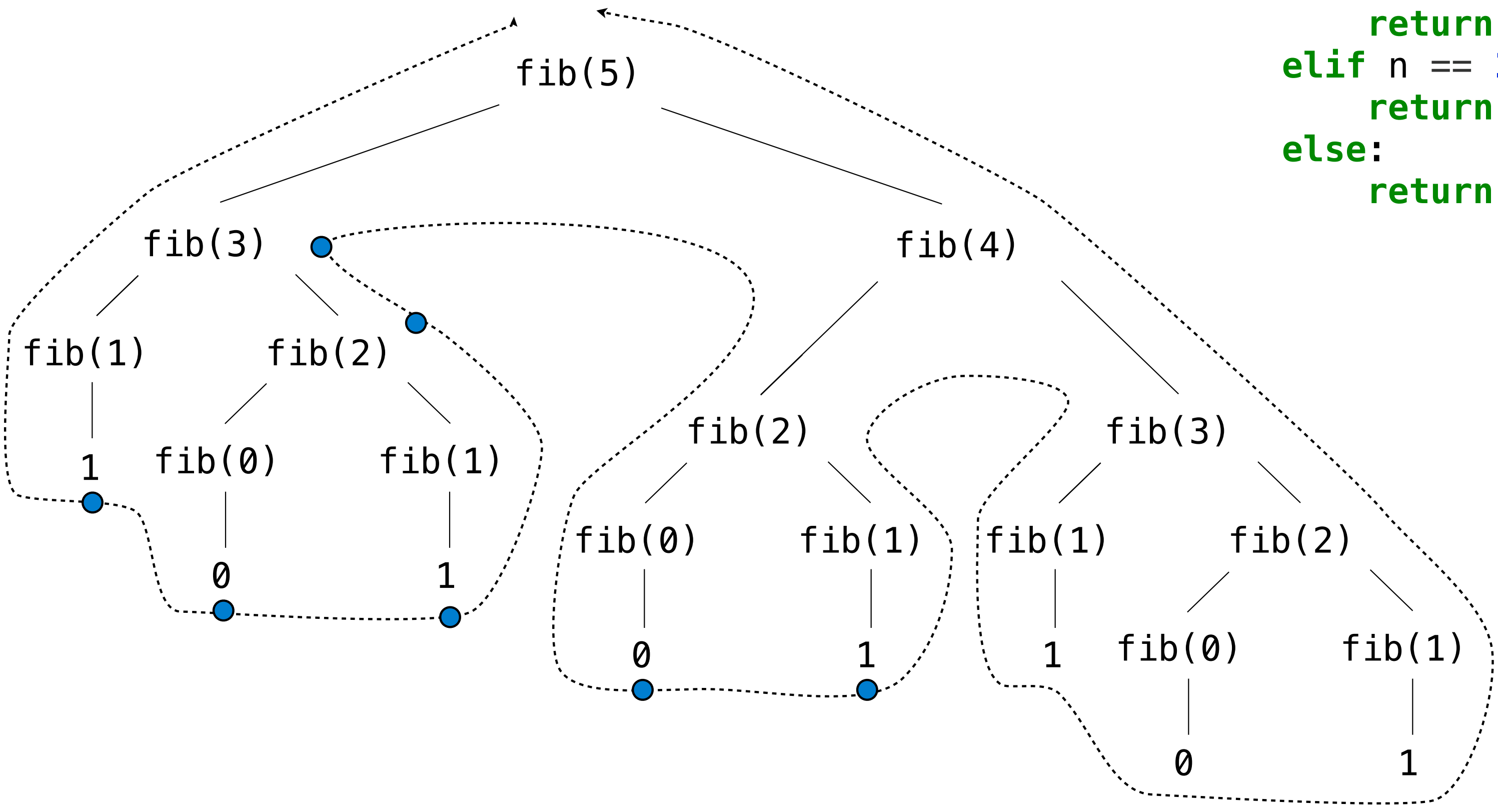


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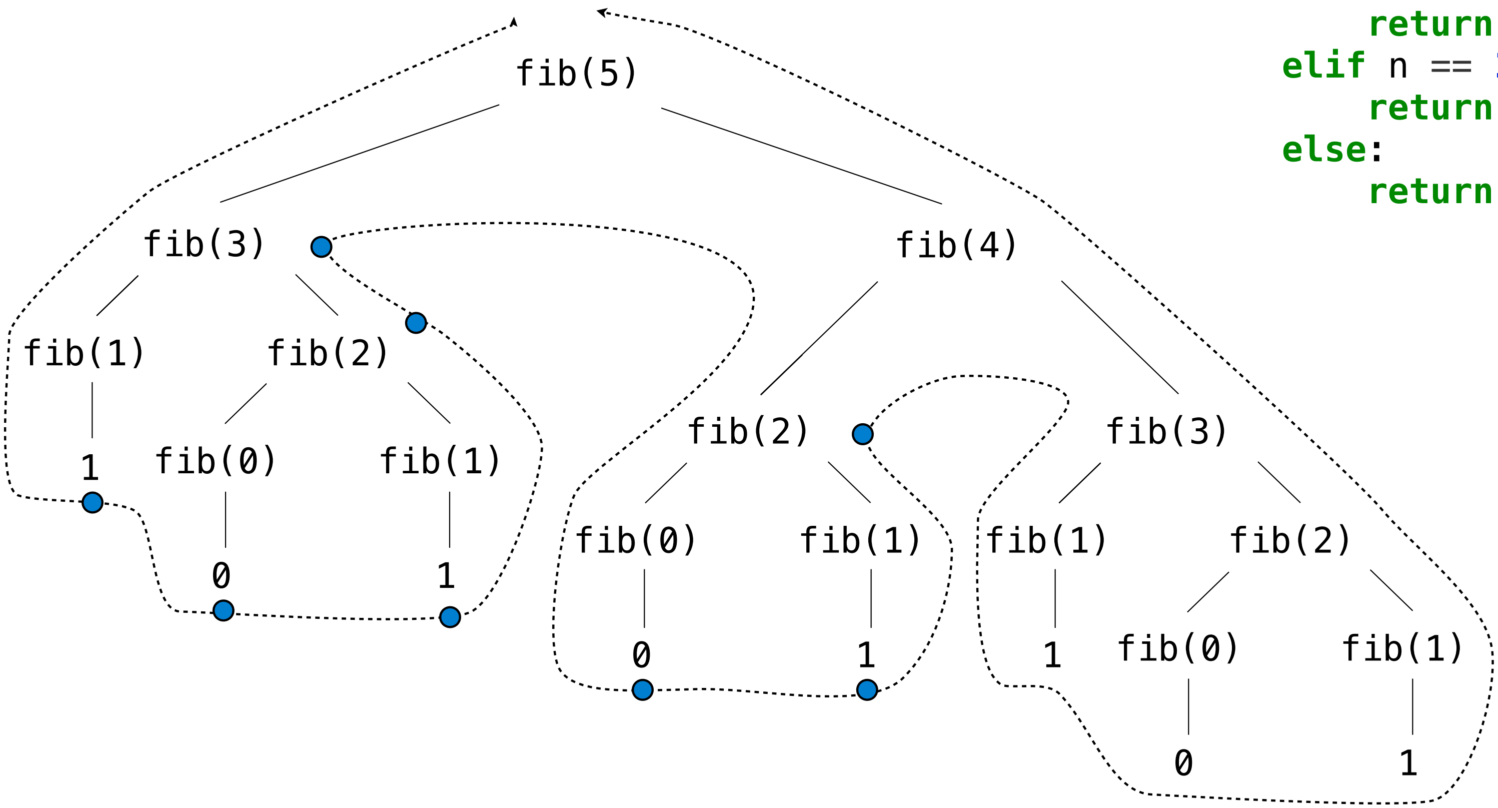

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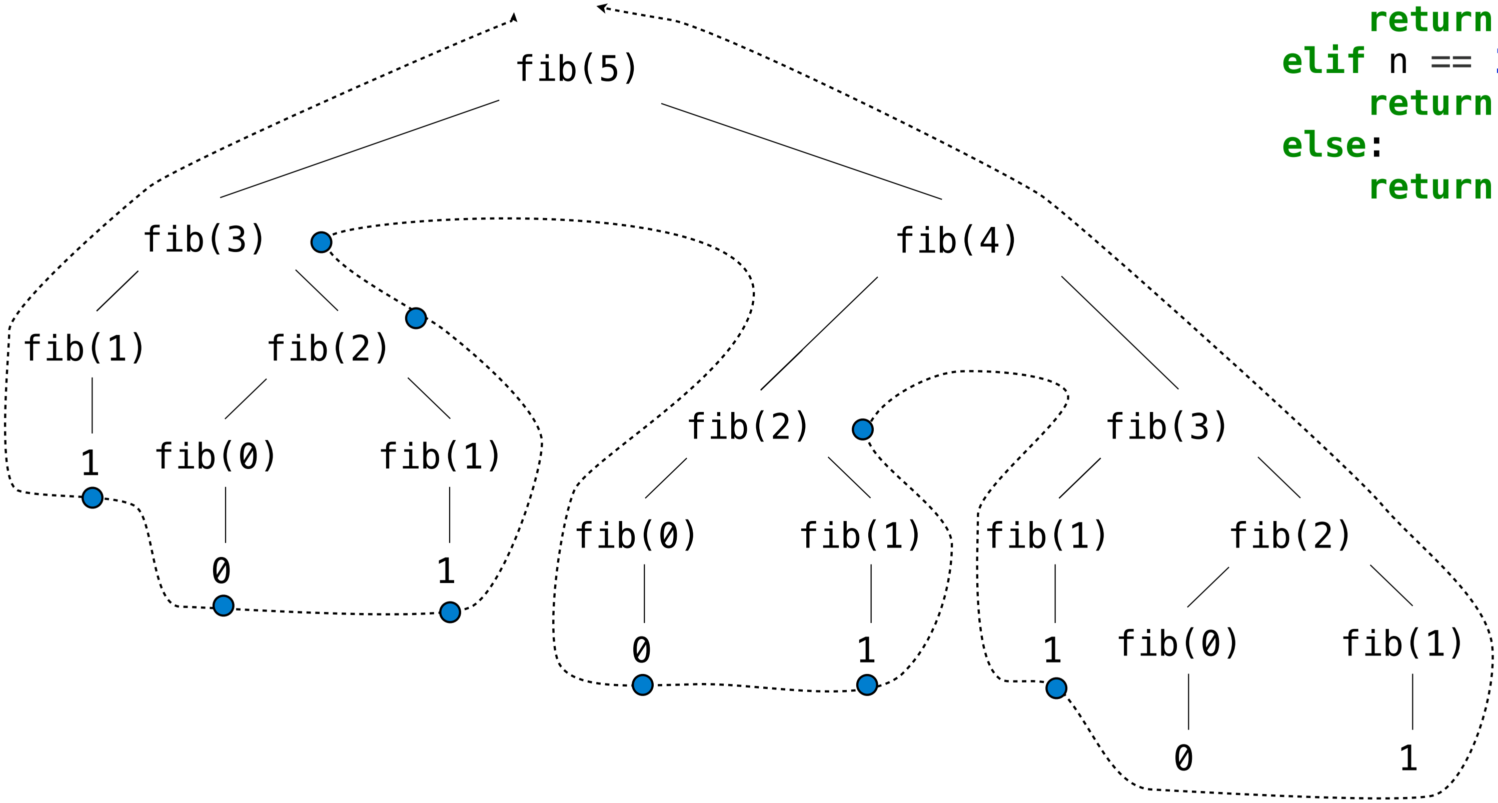

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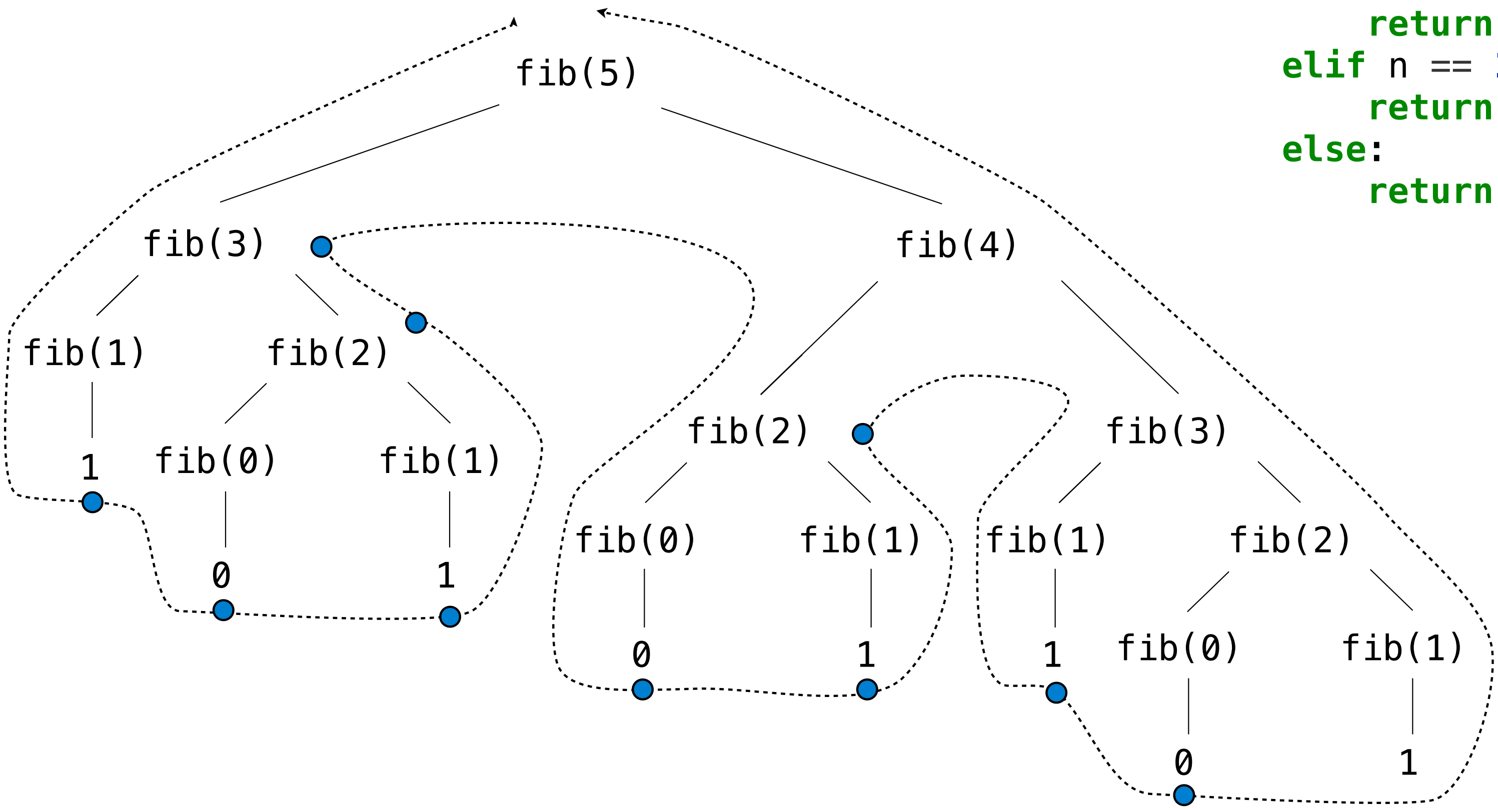

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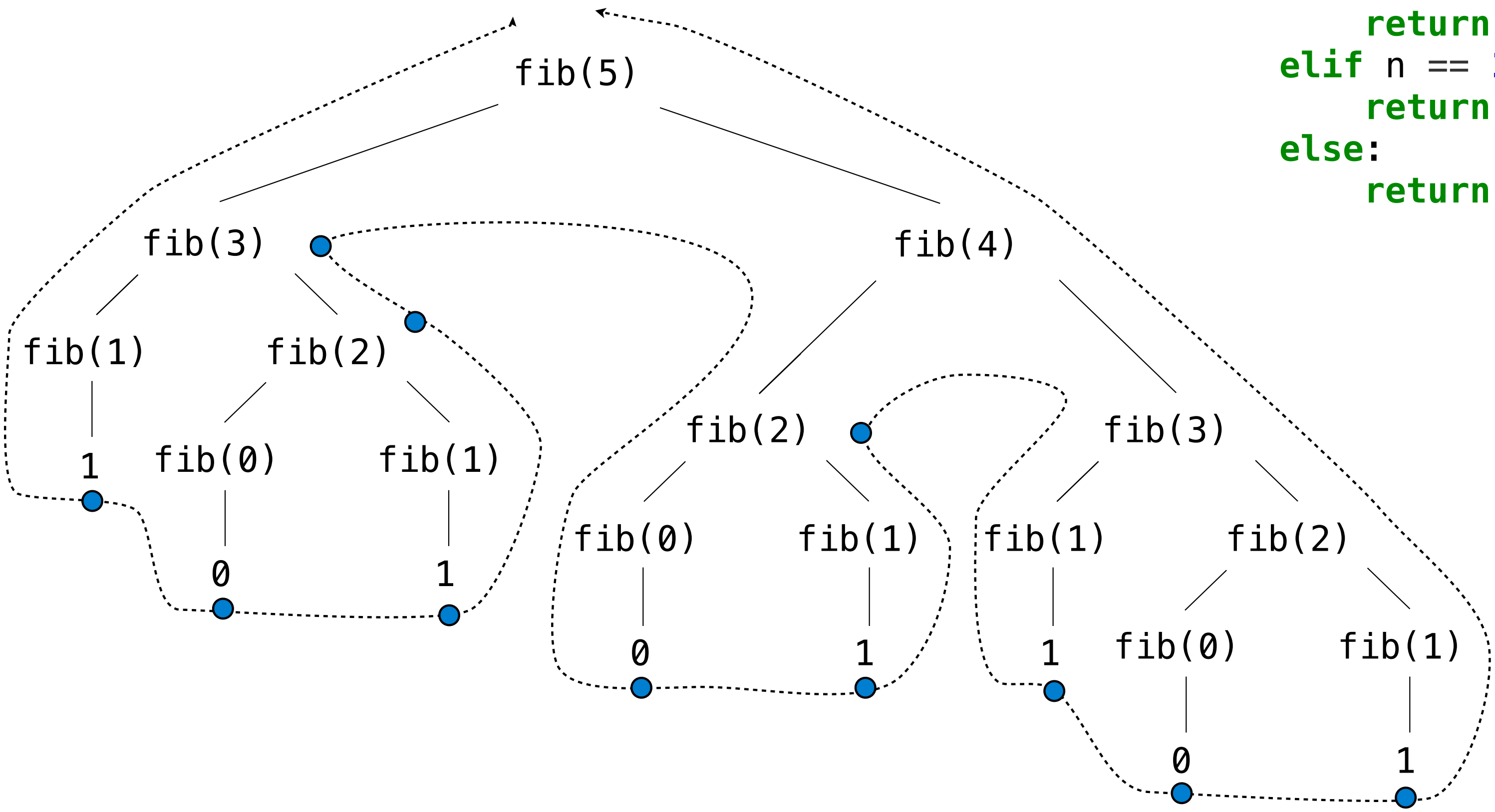

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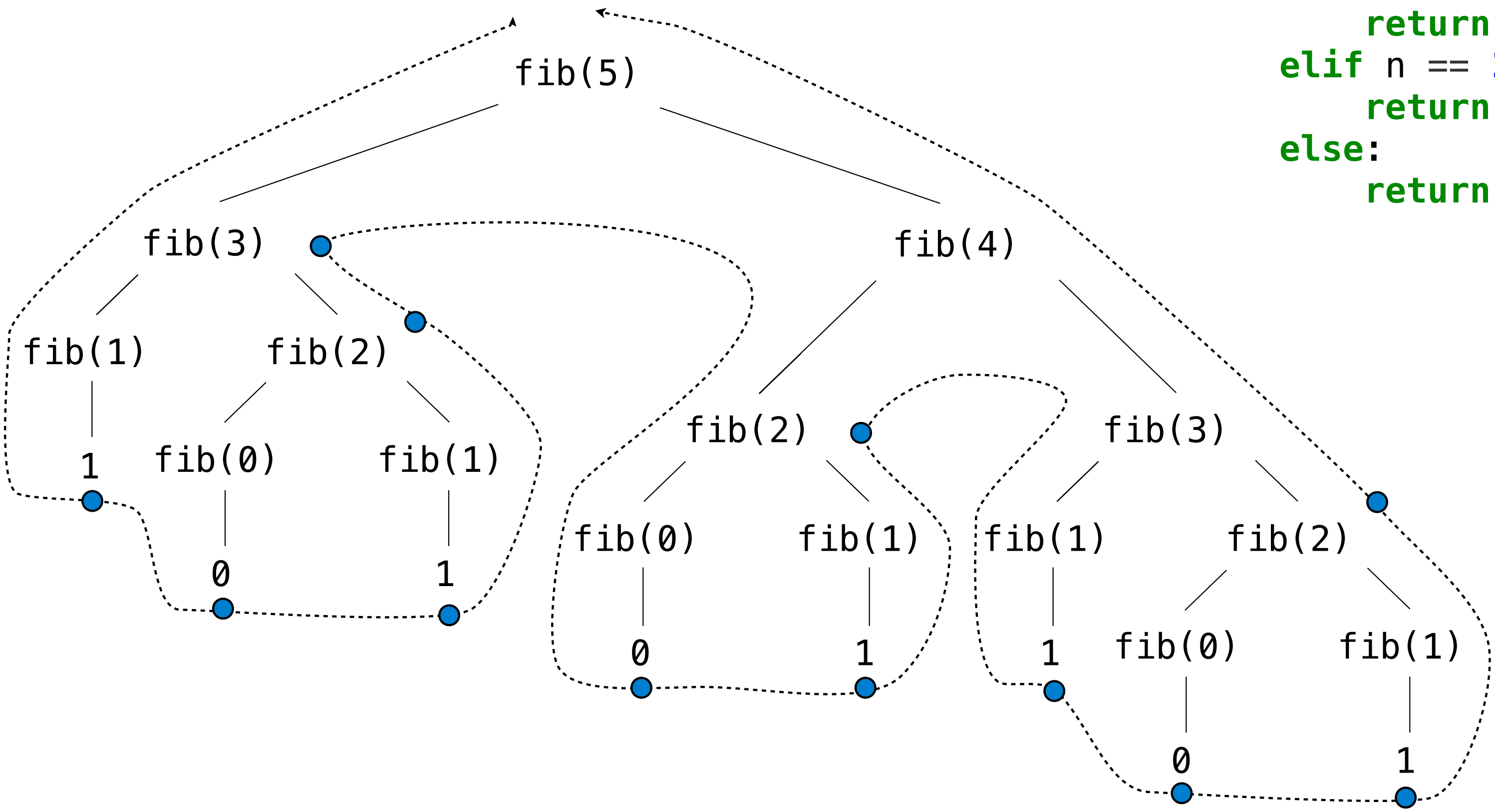

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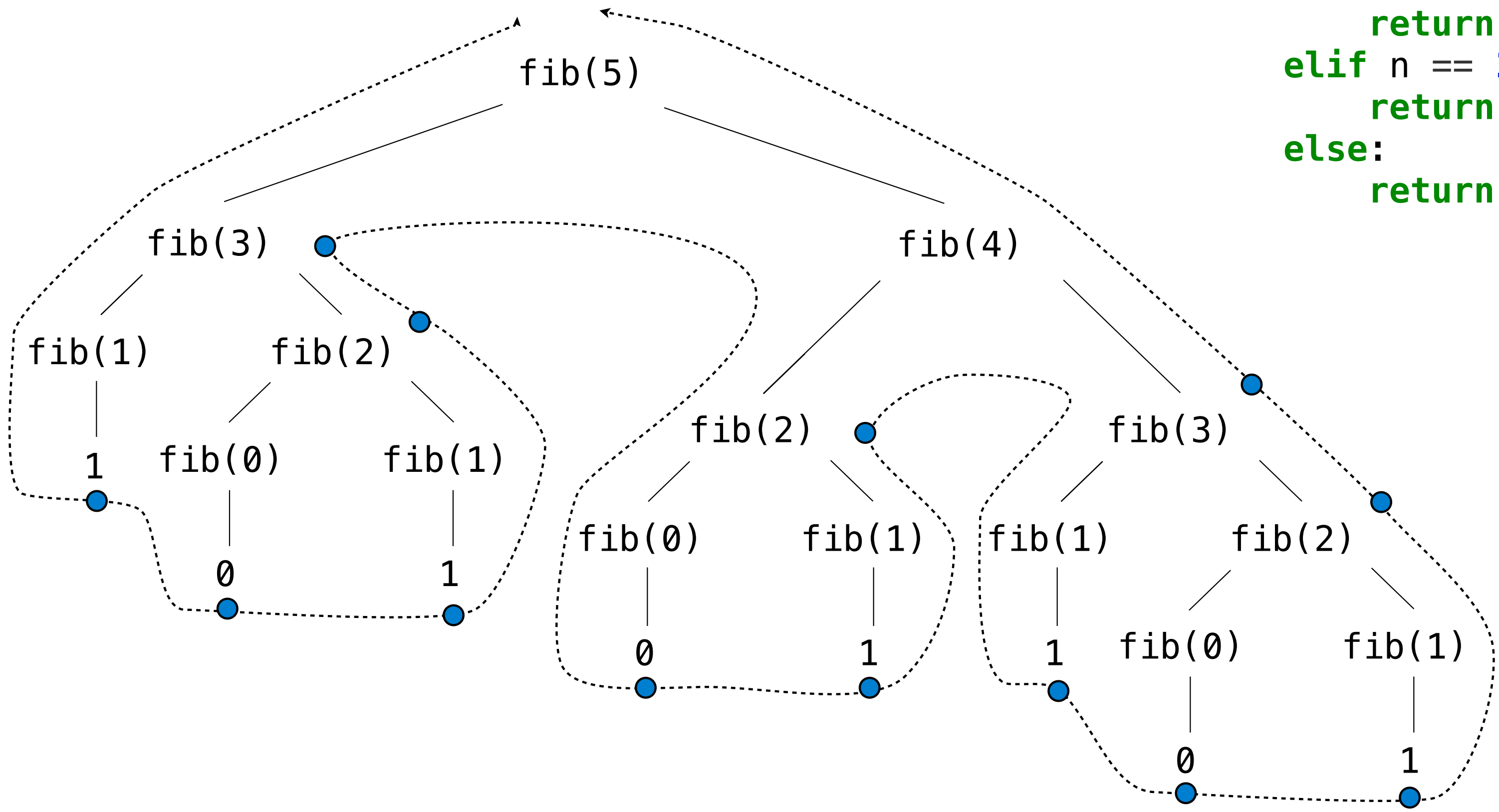

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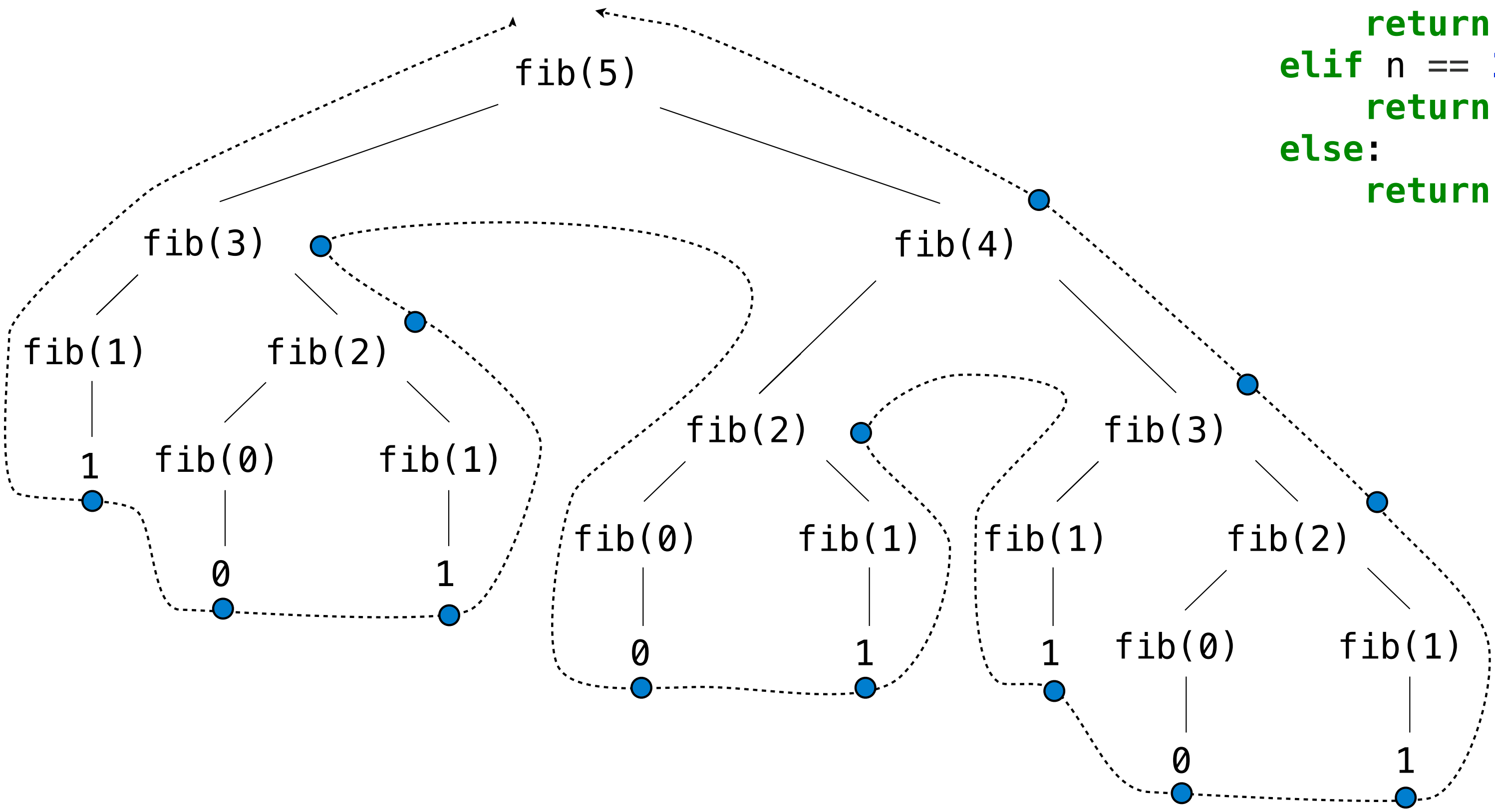

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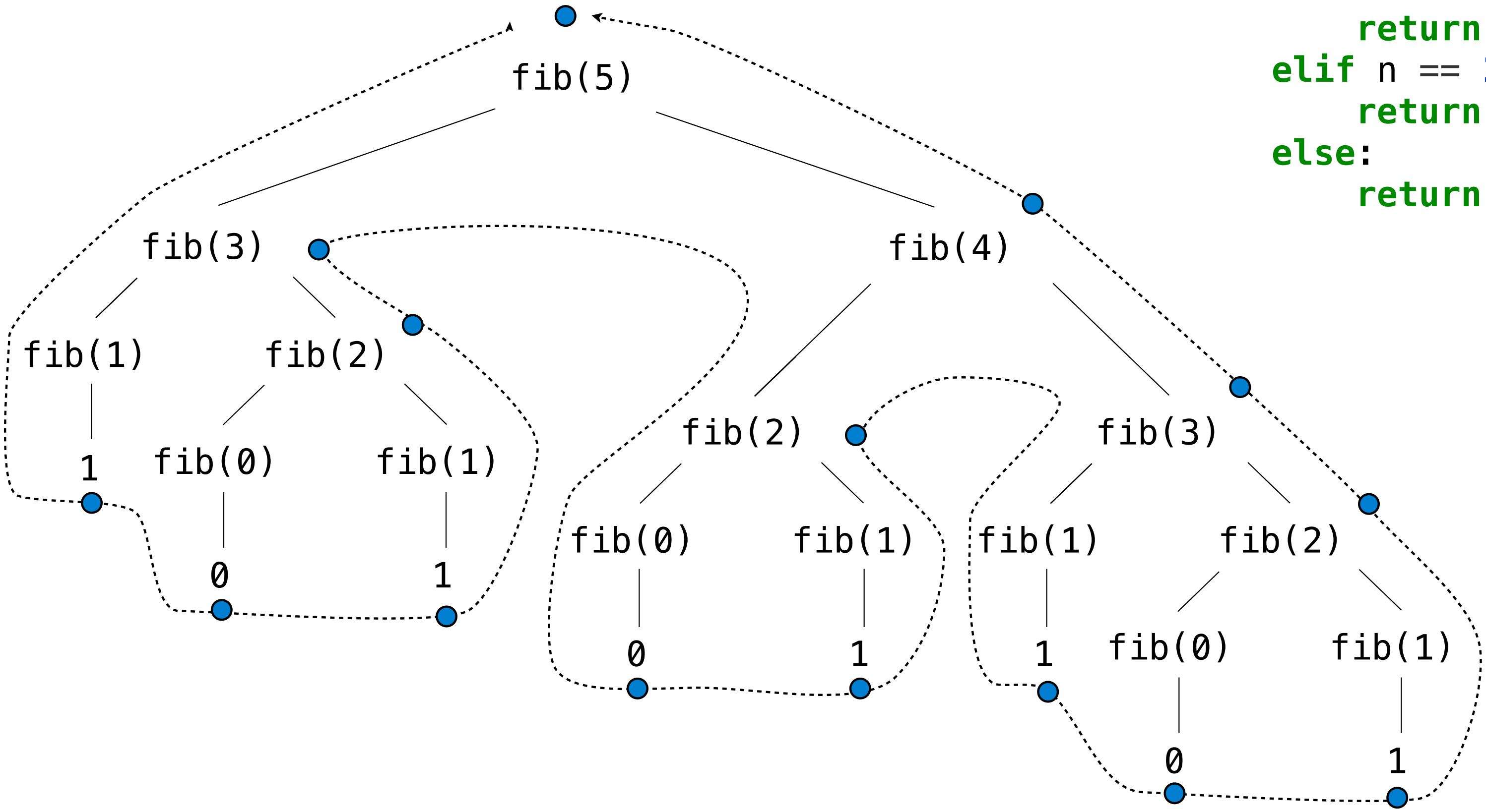

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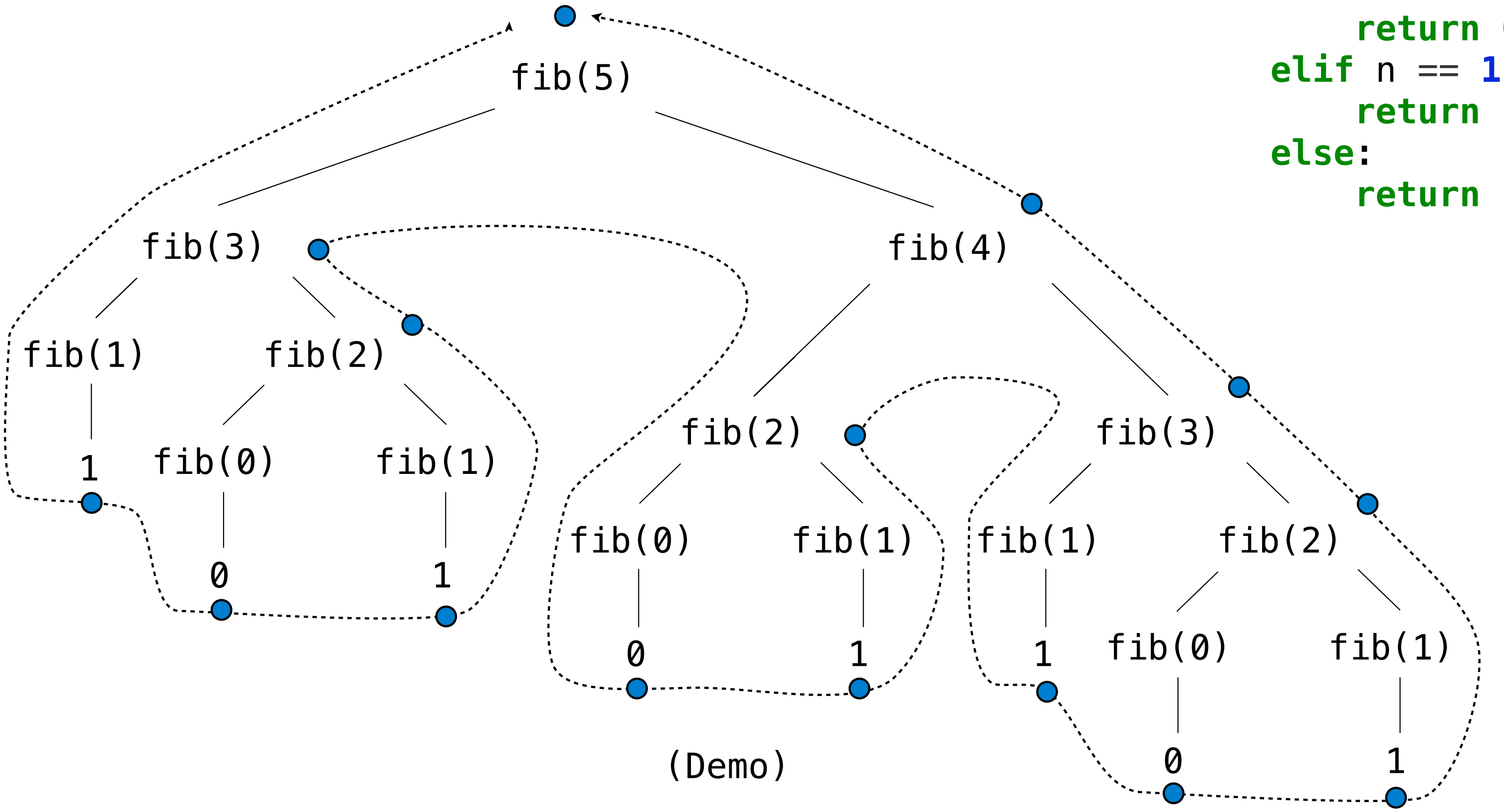

(Demo)

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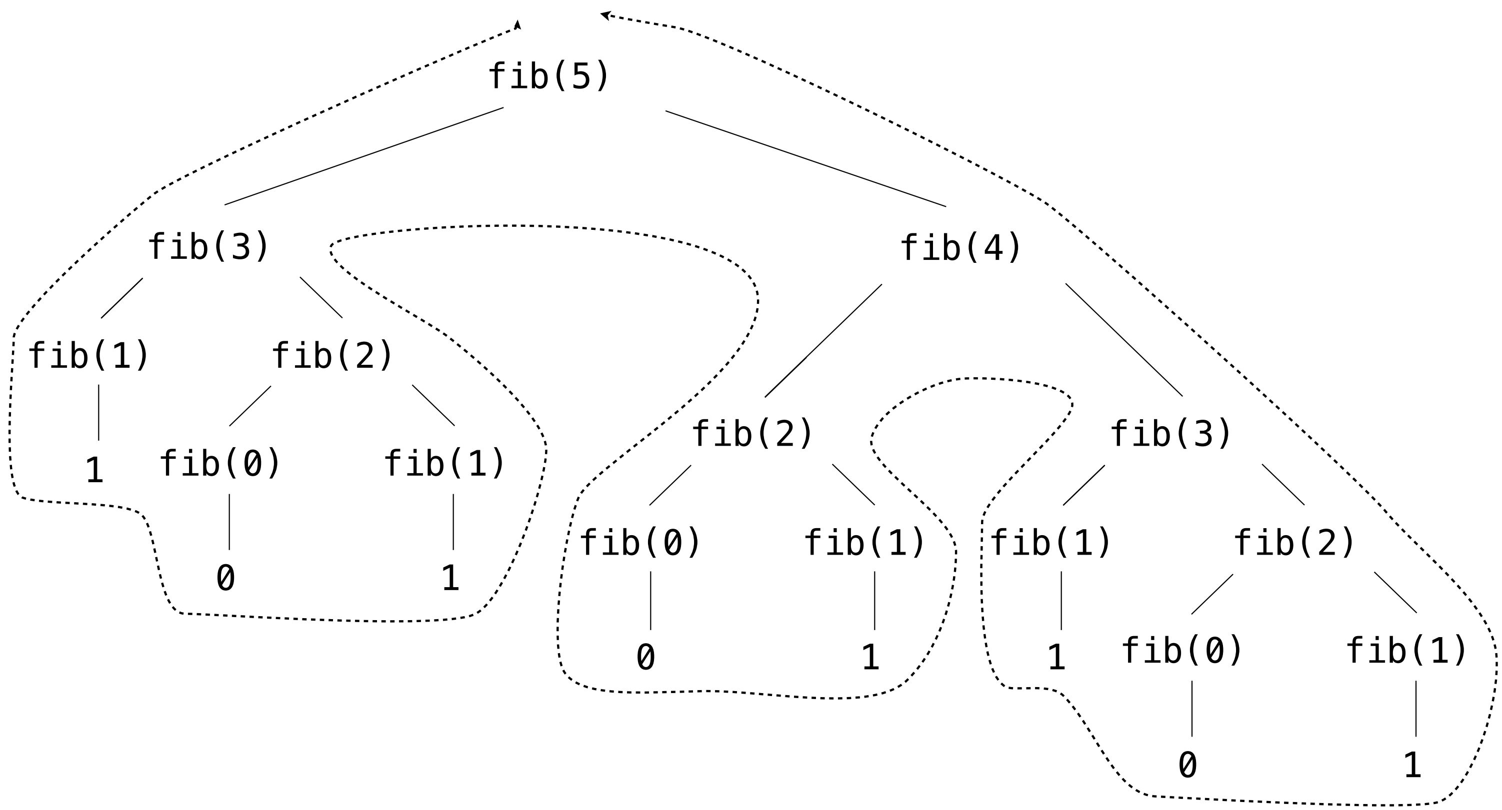

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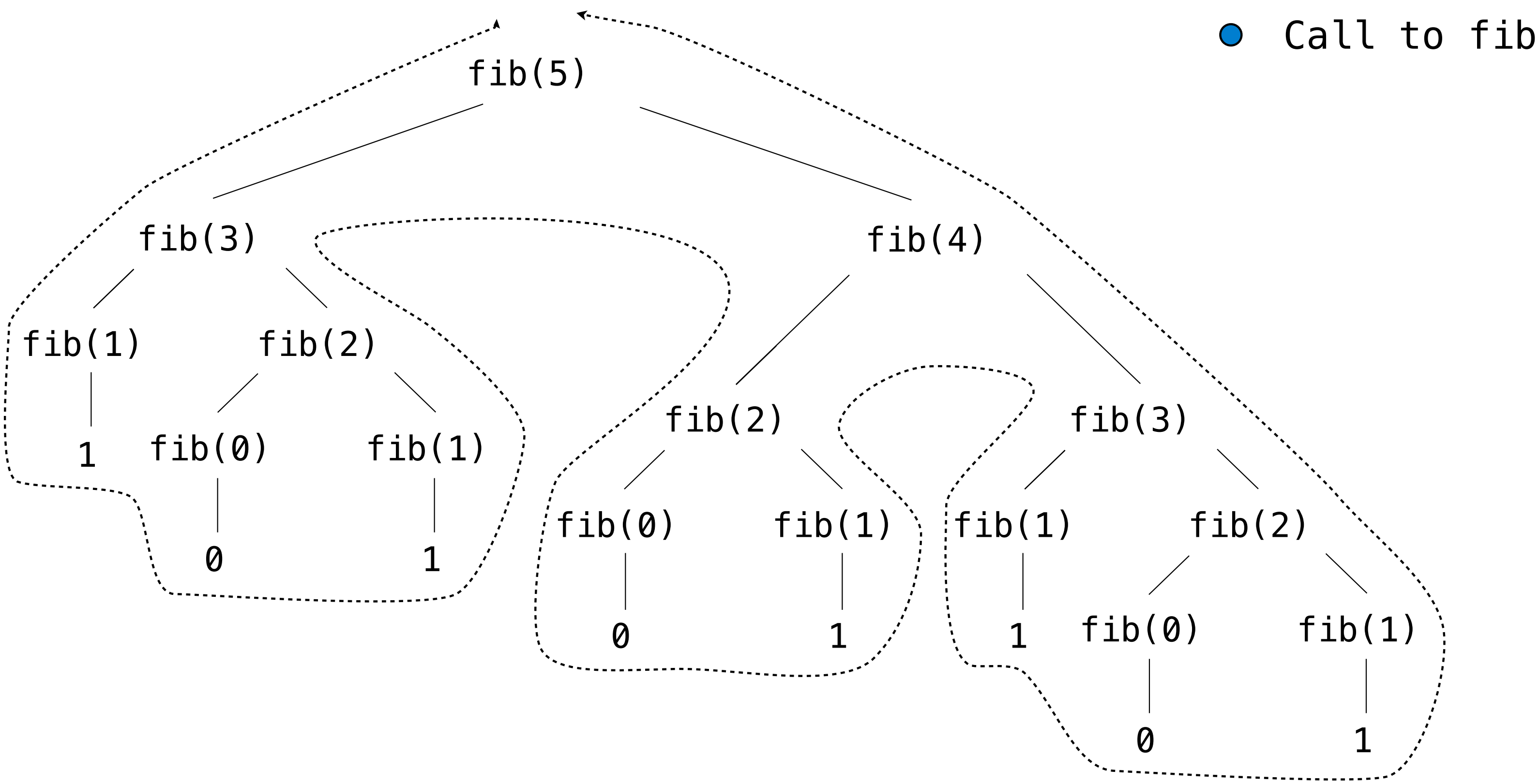

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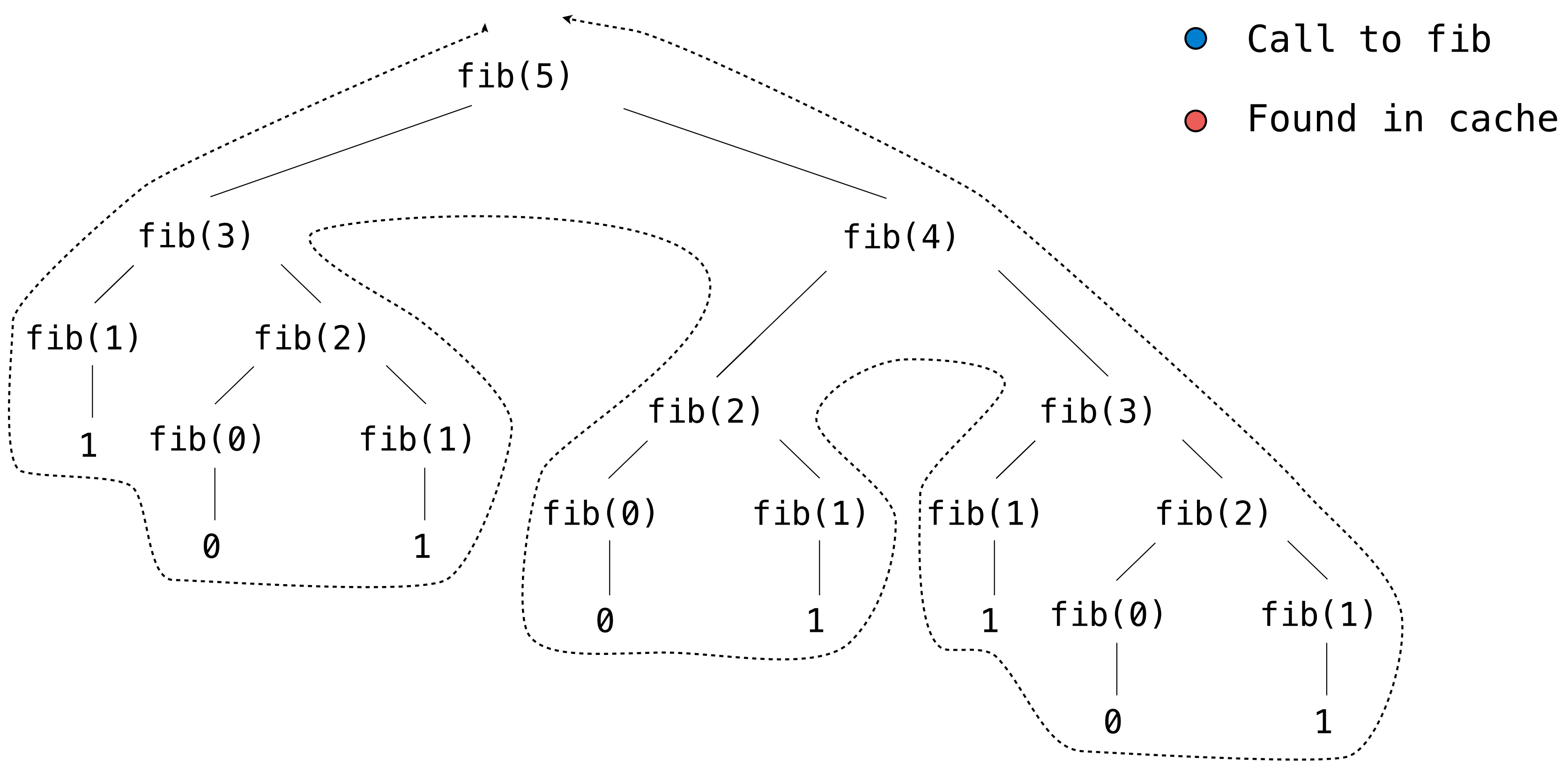

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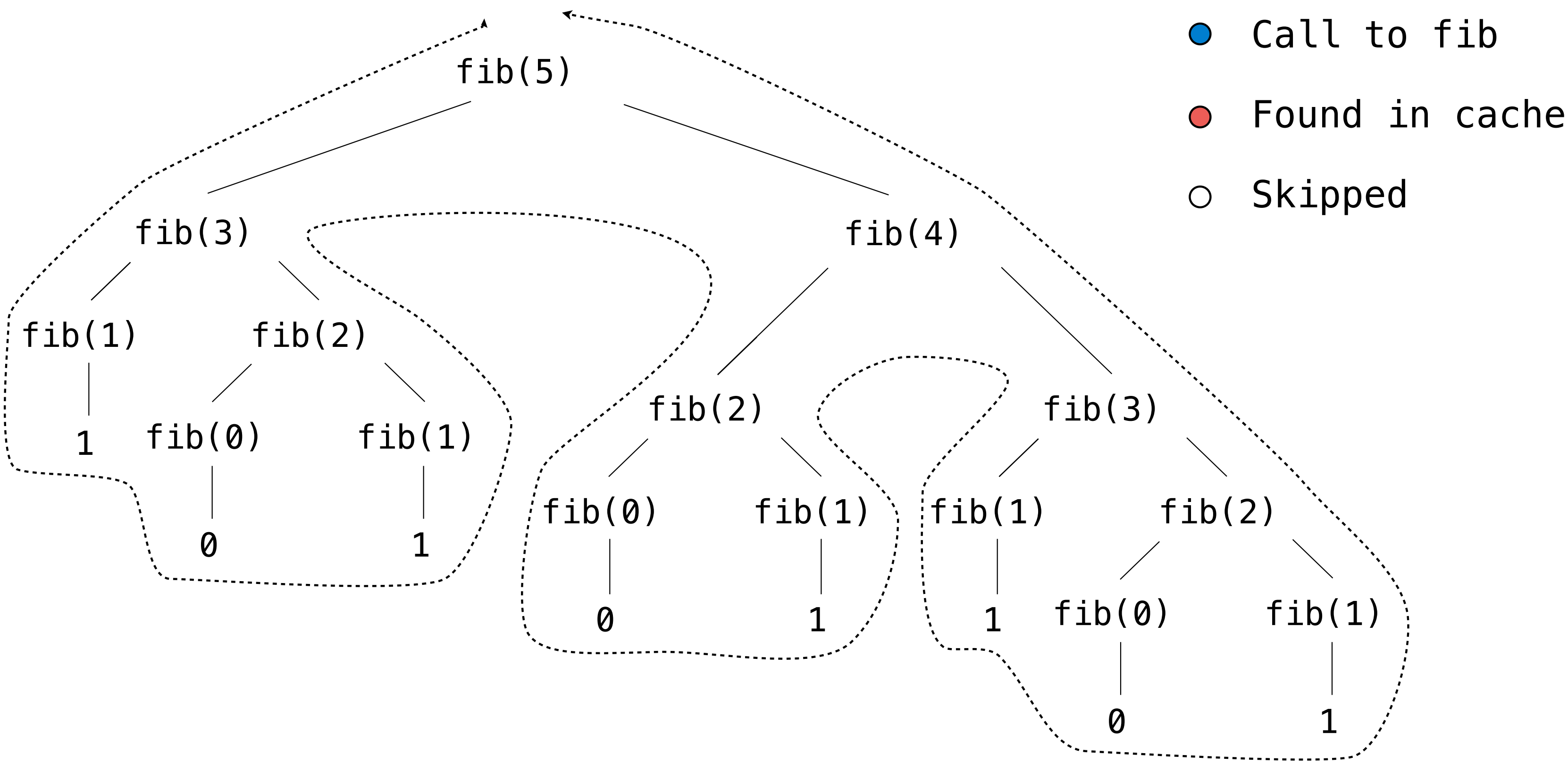

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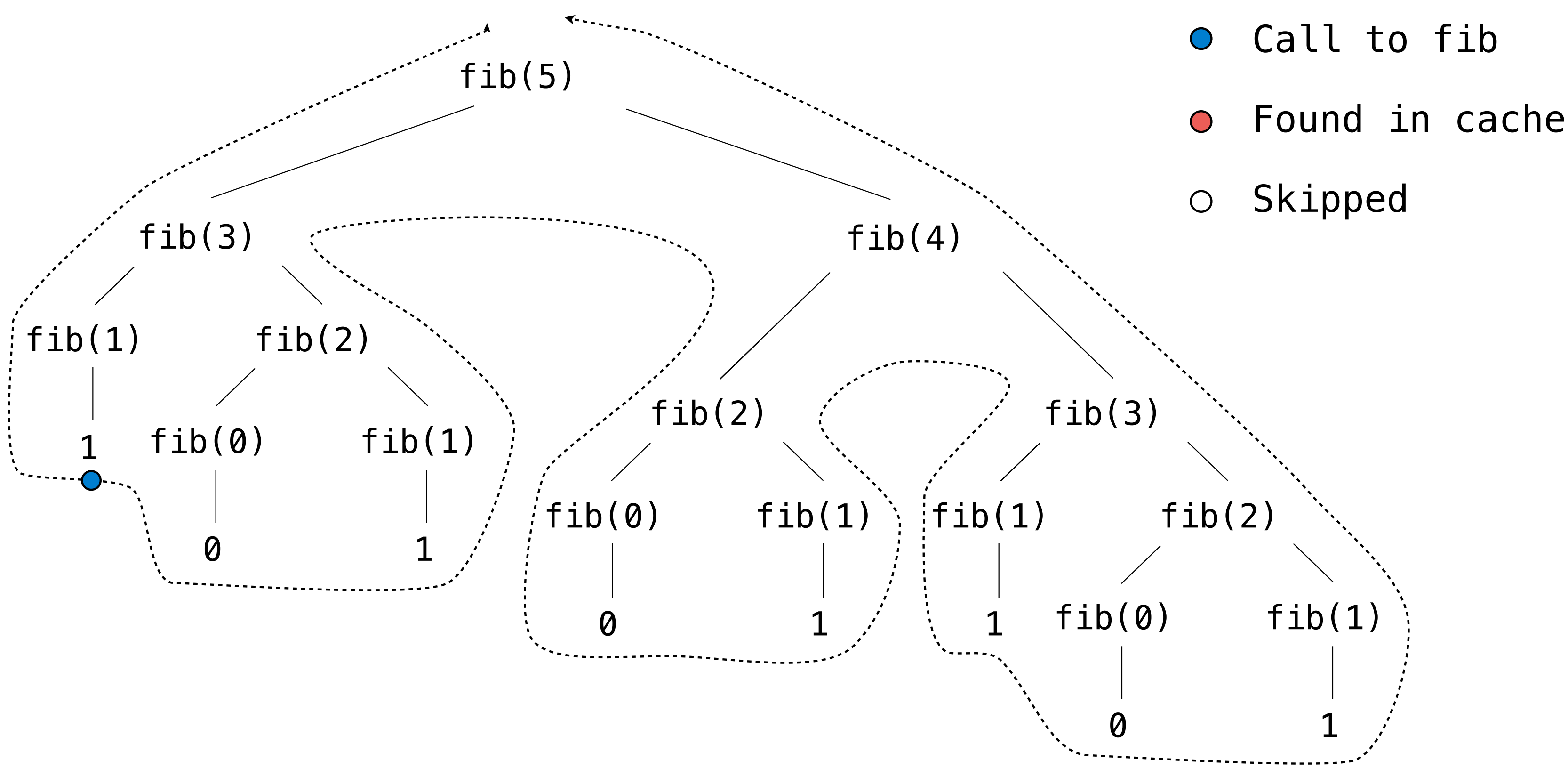

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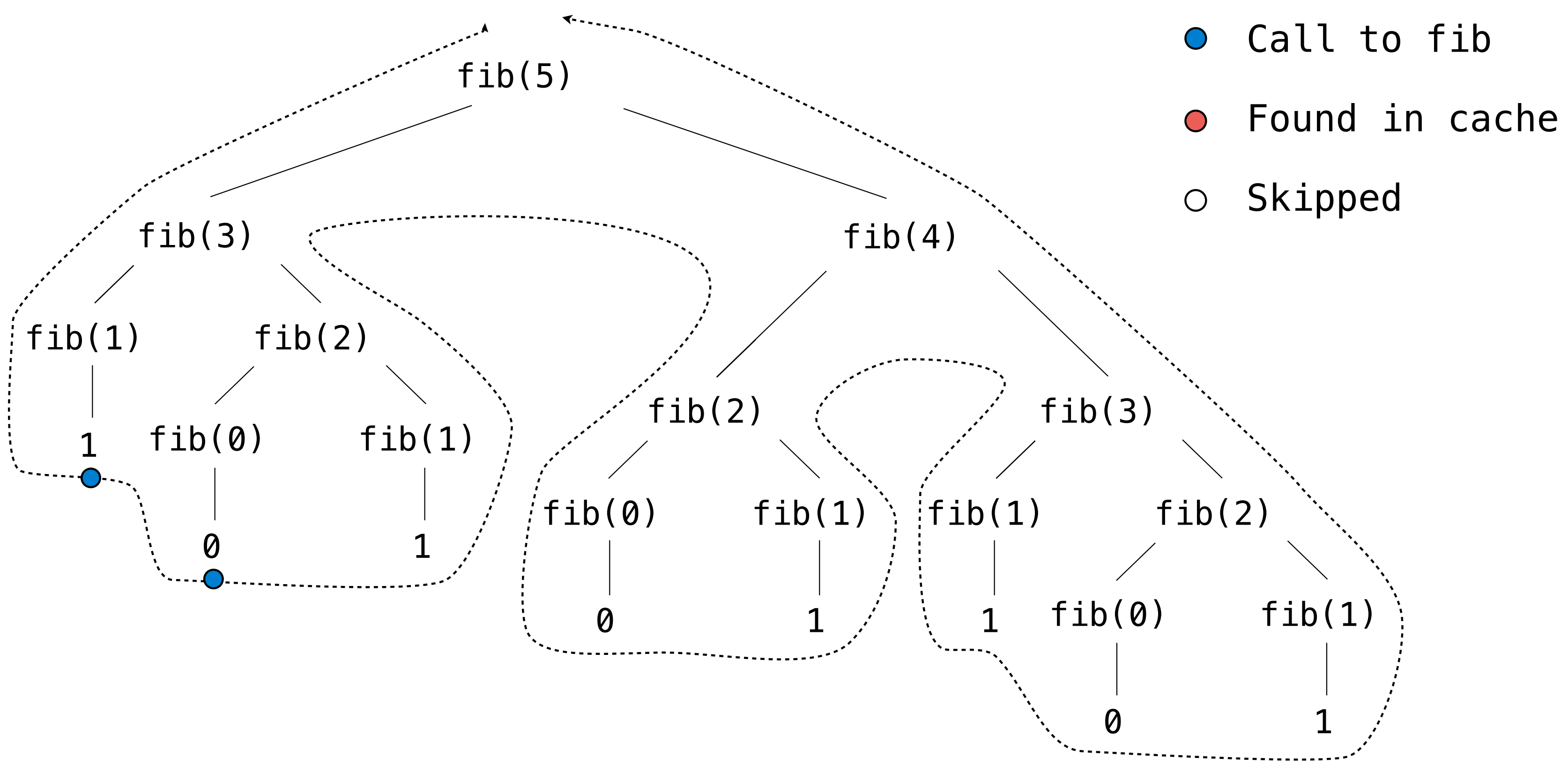

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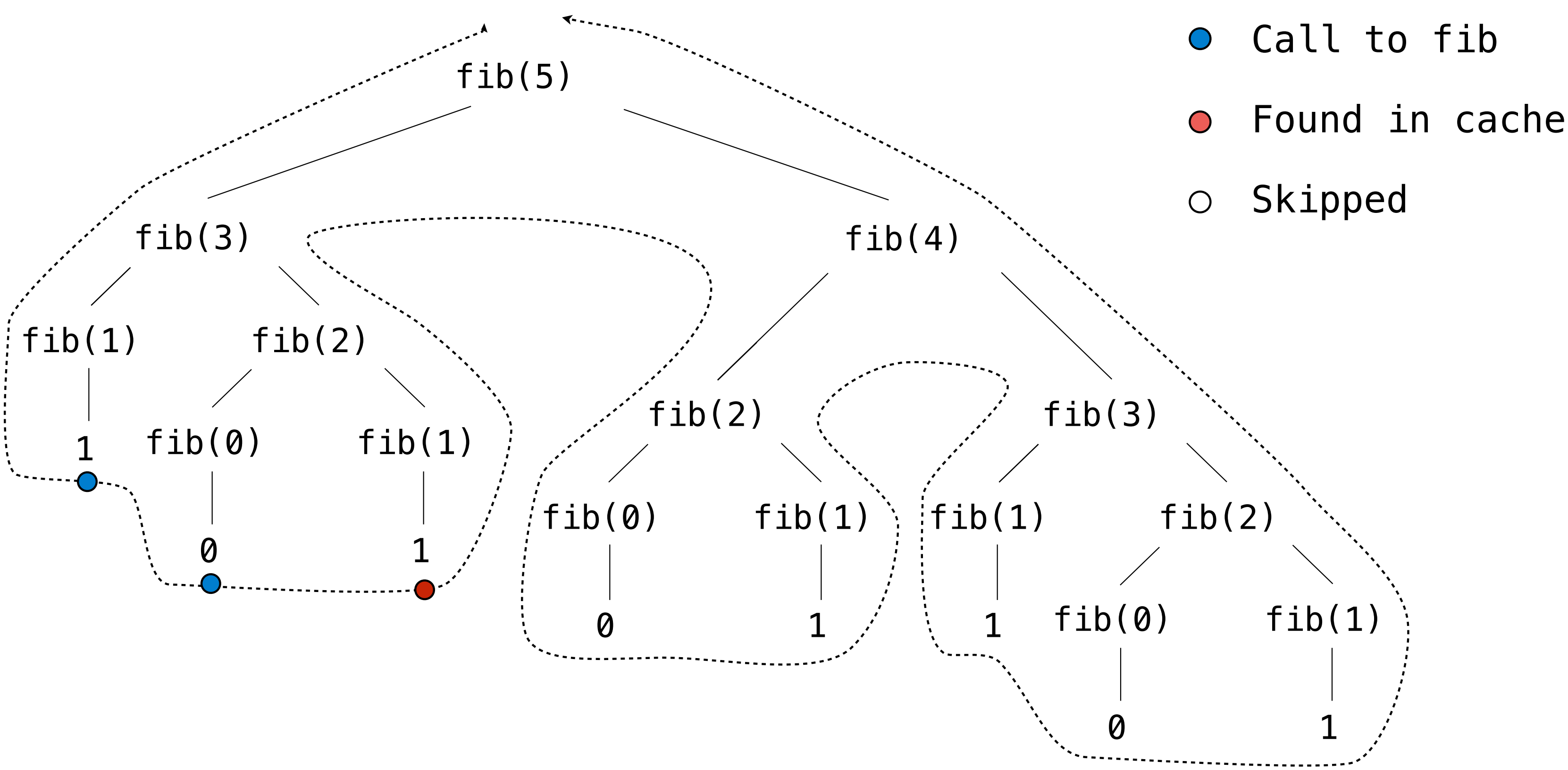

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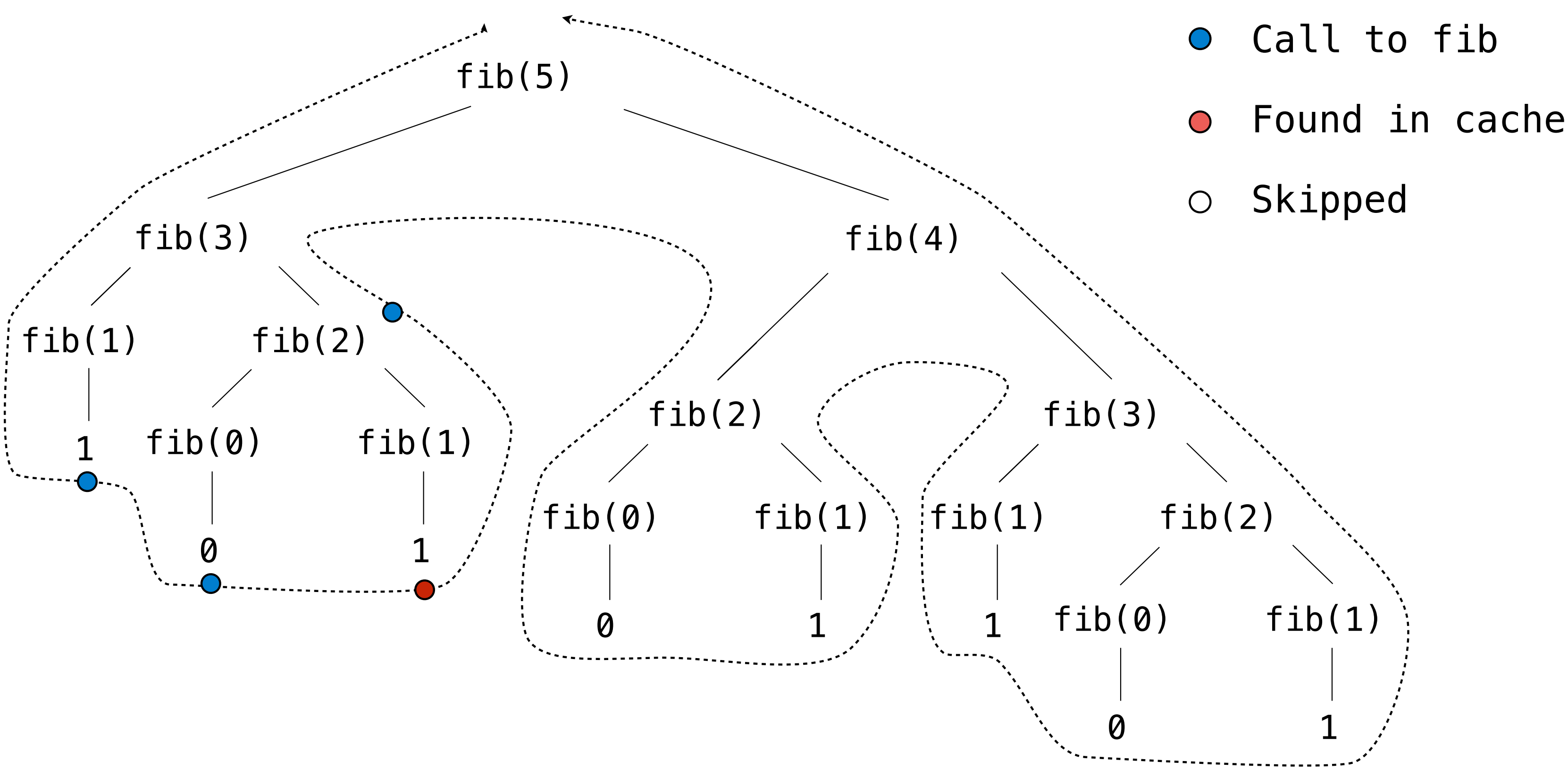

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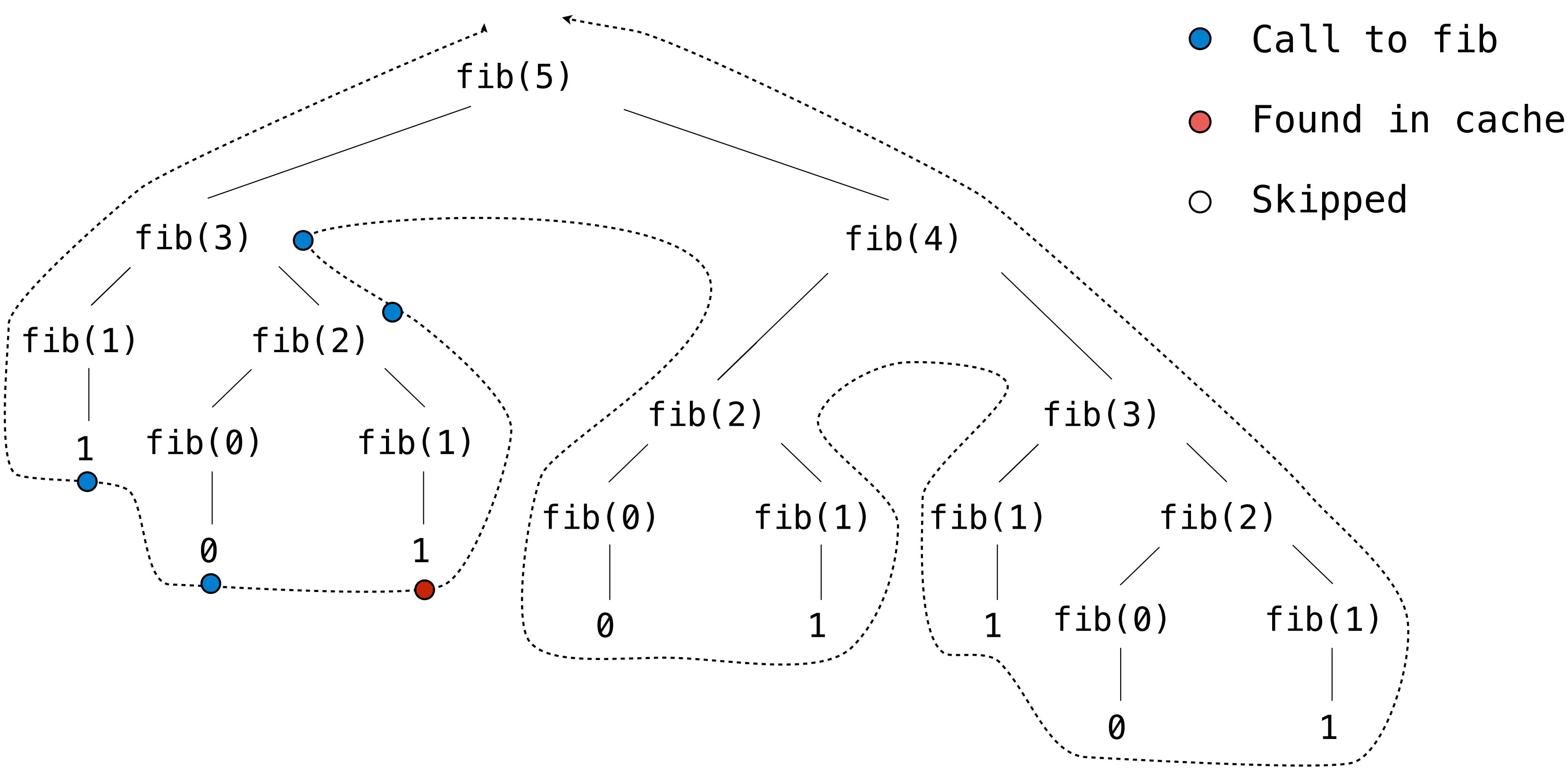

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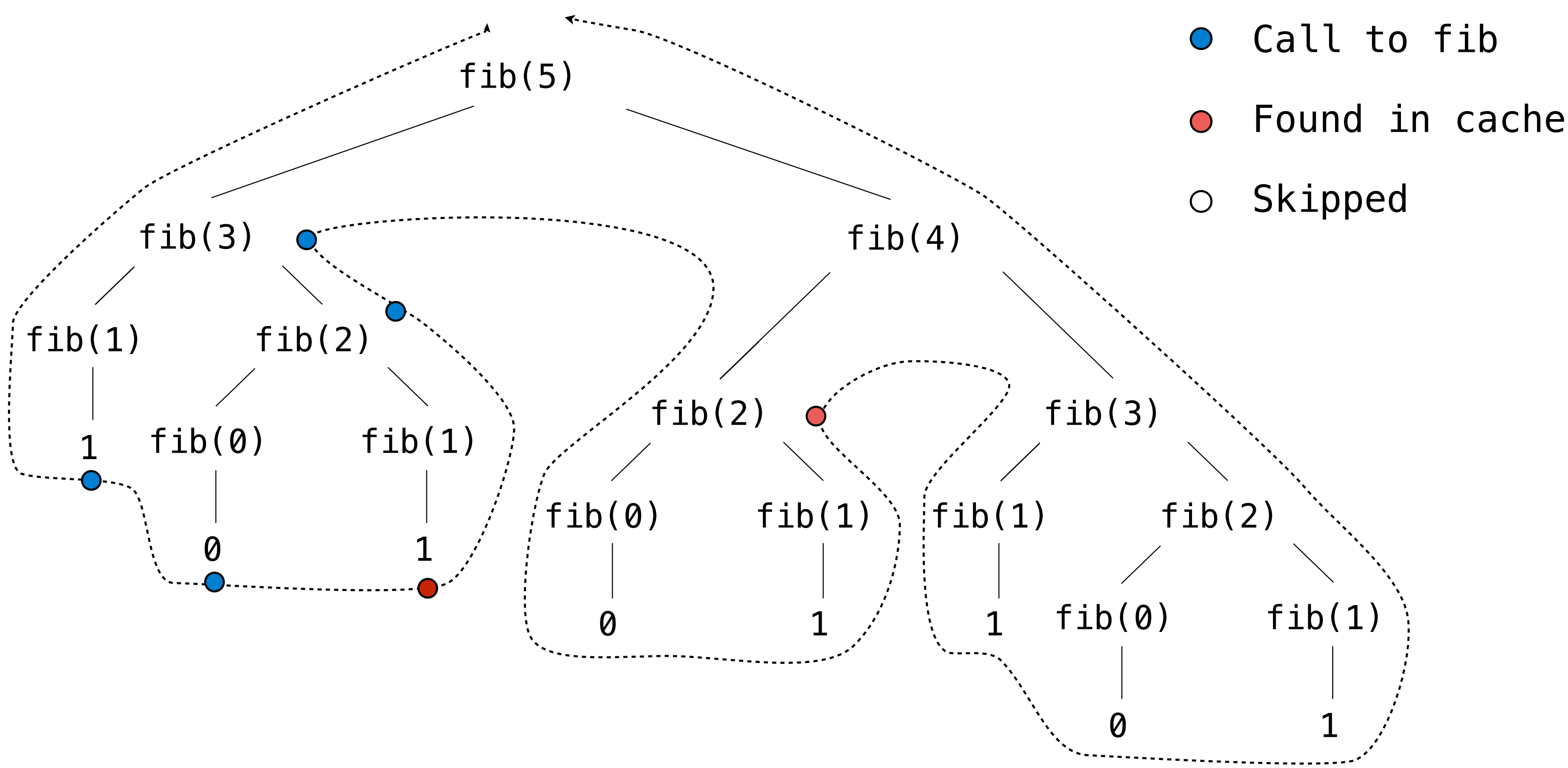

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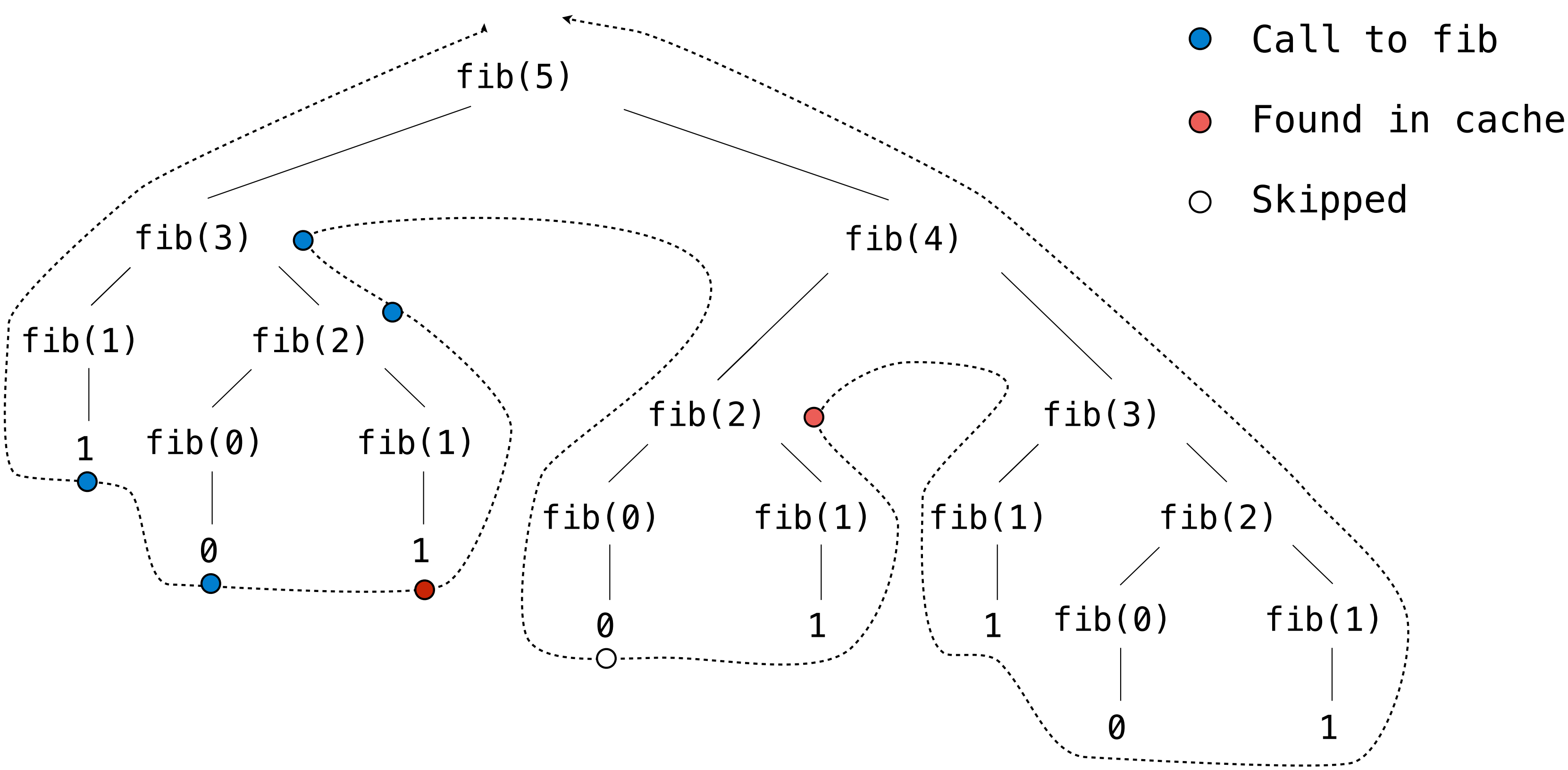

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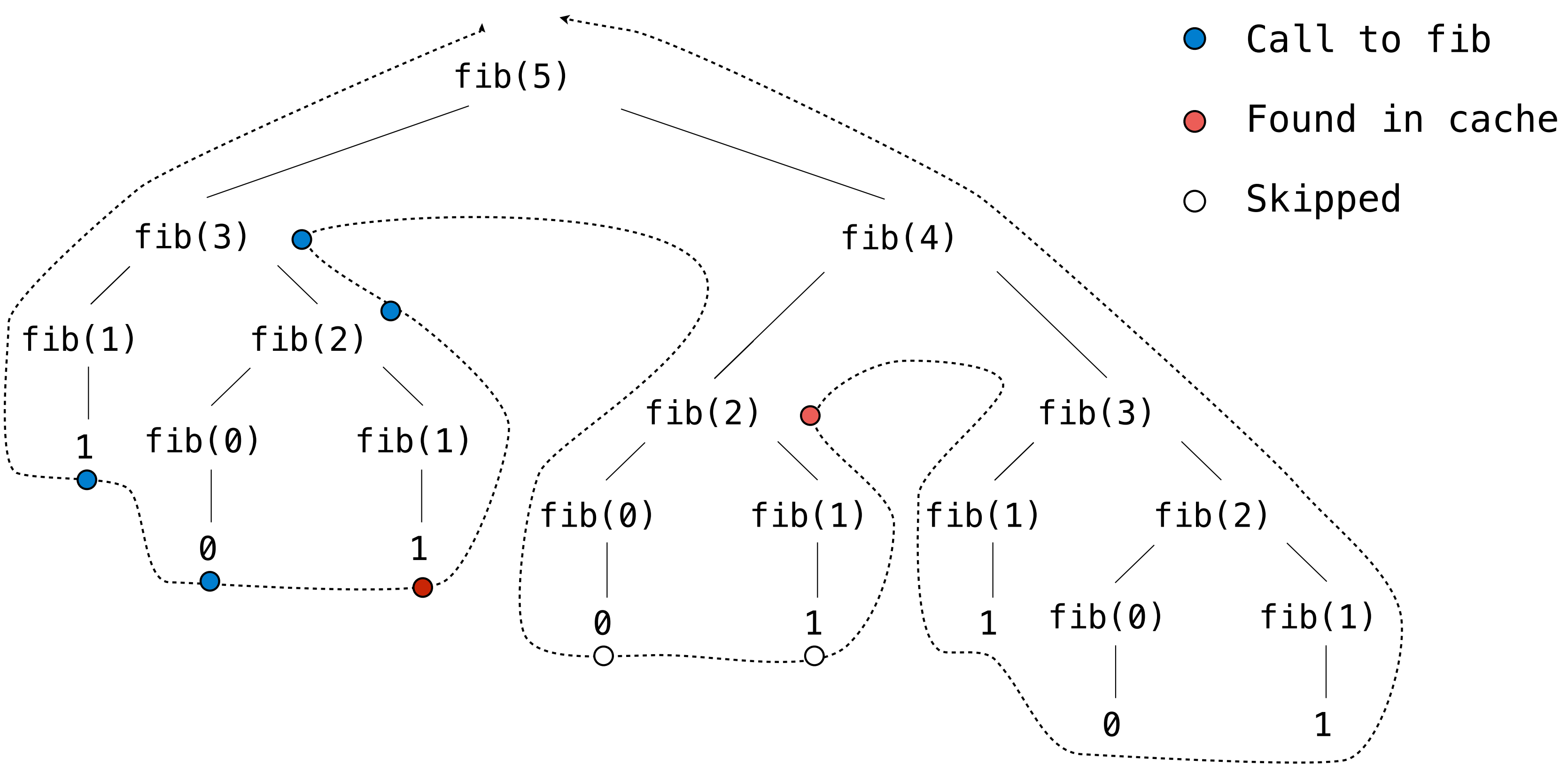

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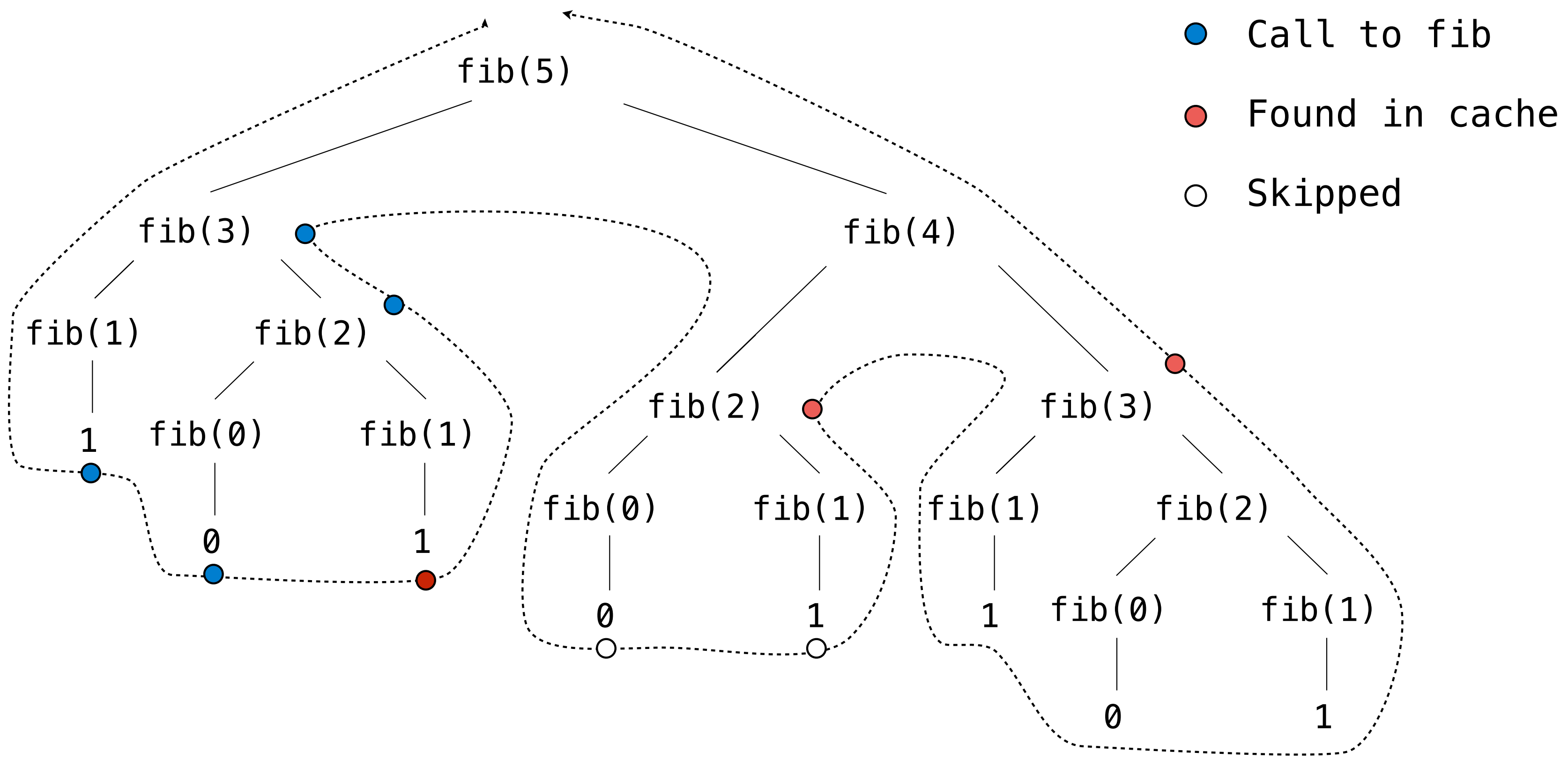

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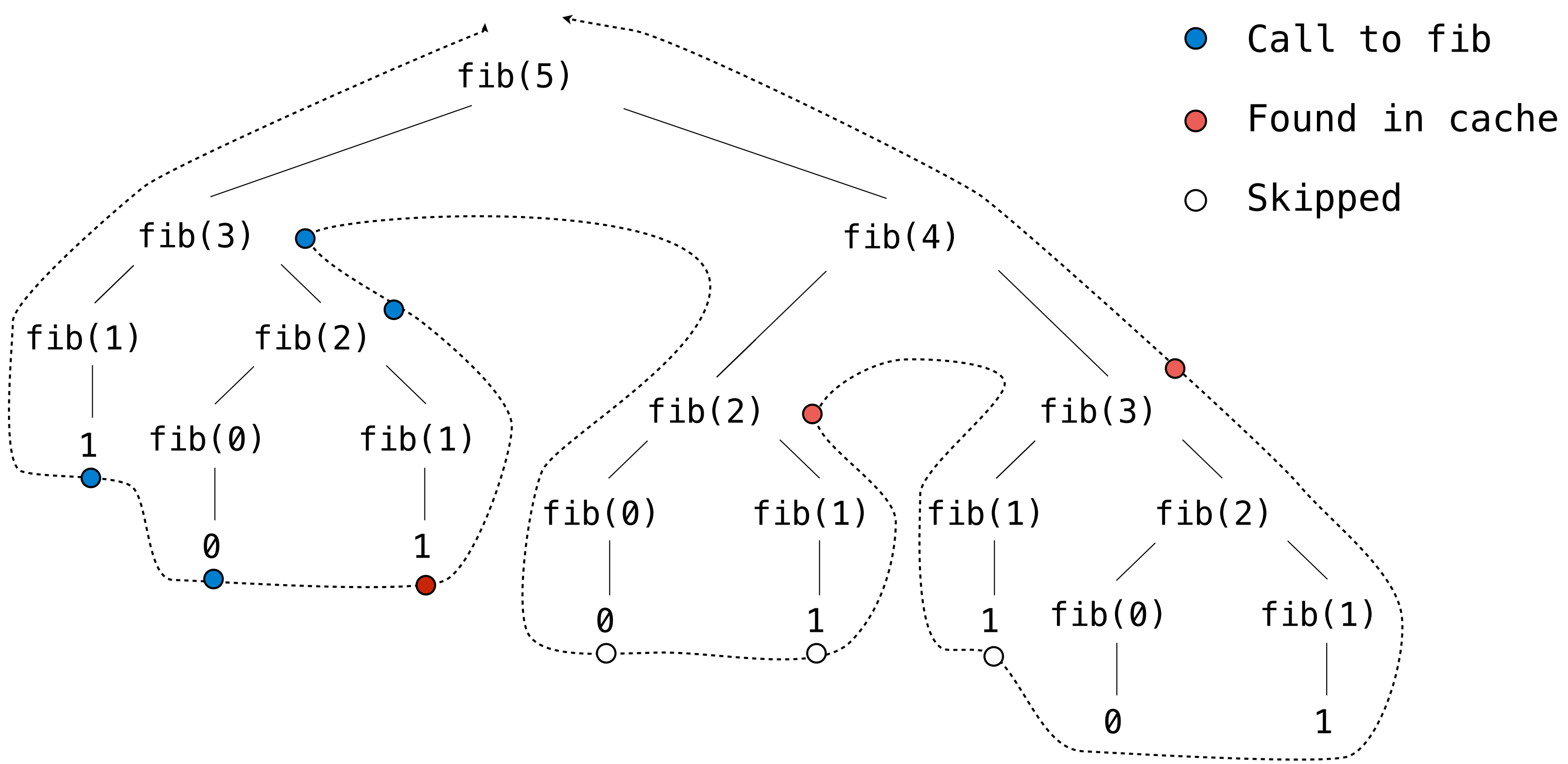

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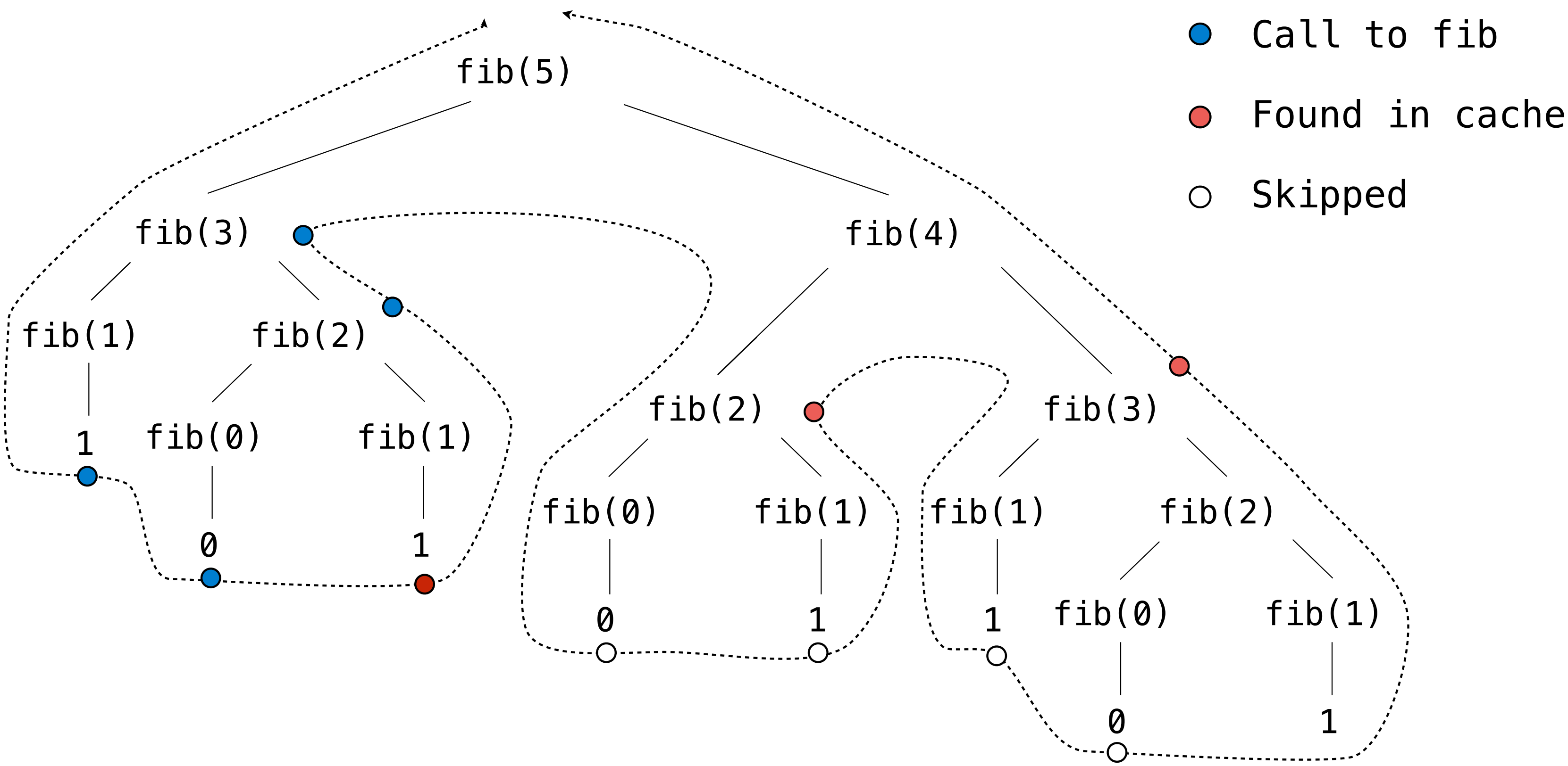

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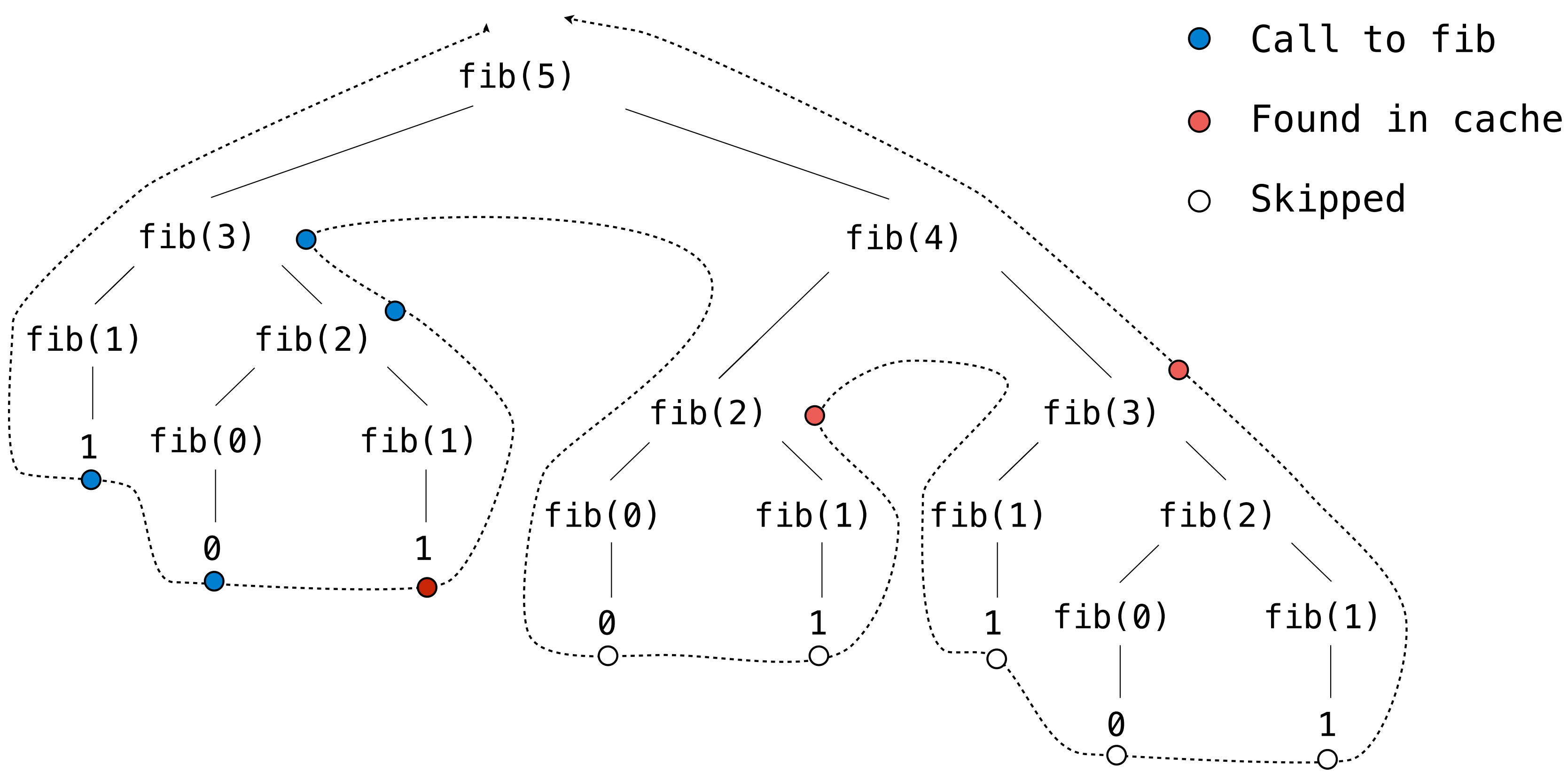

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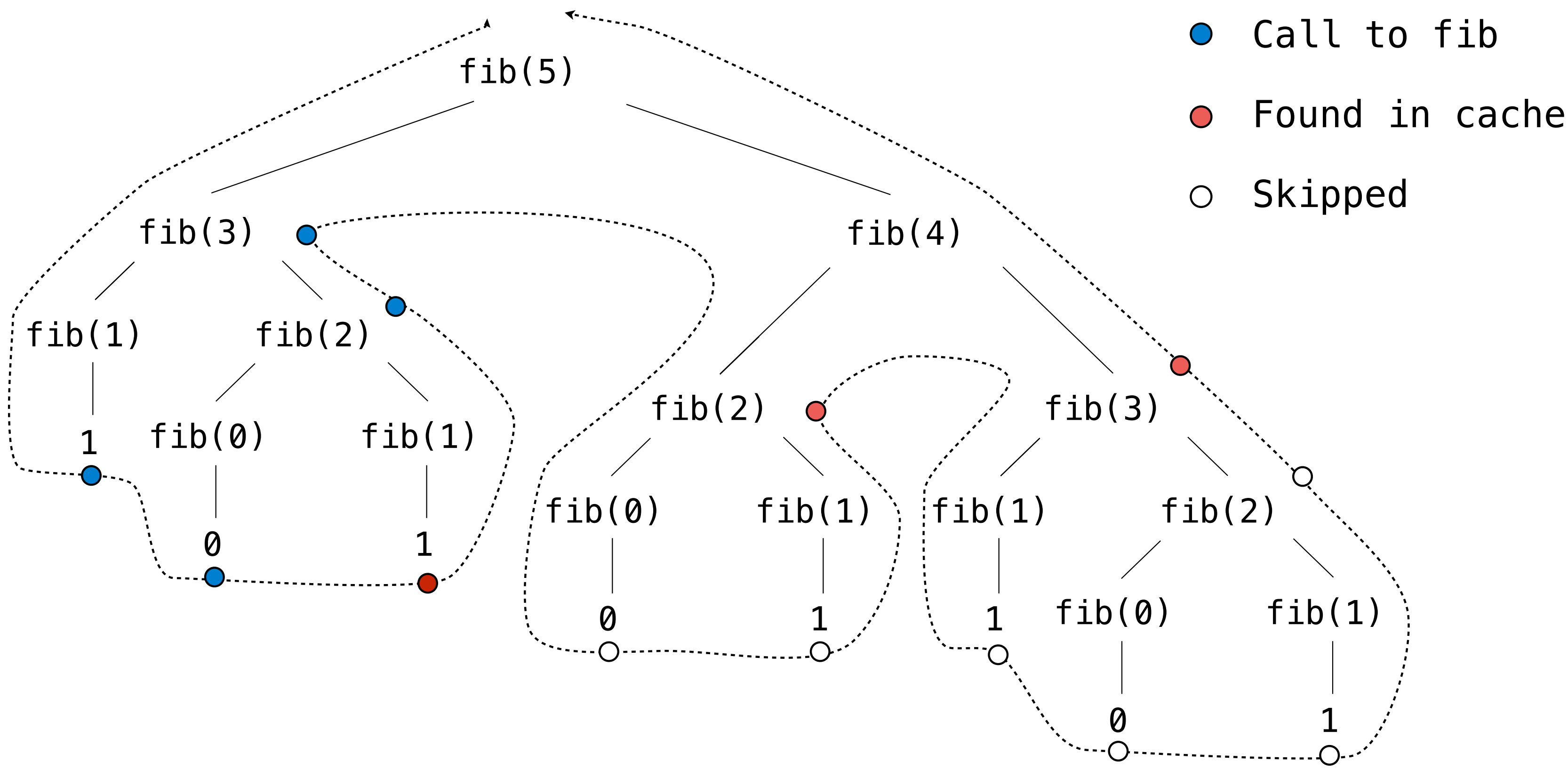

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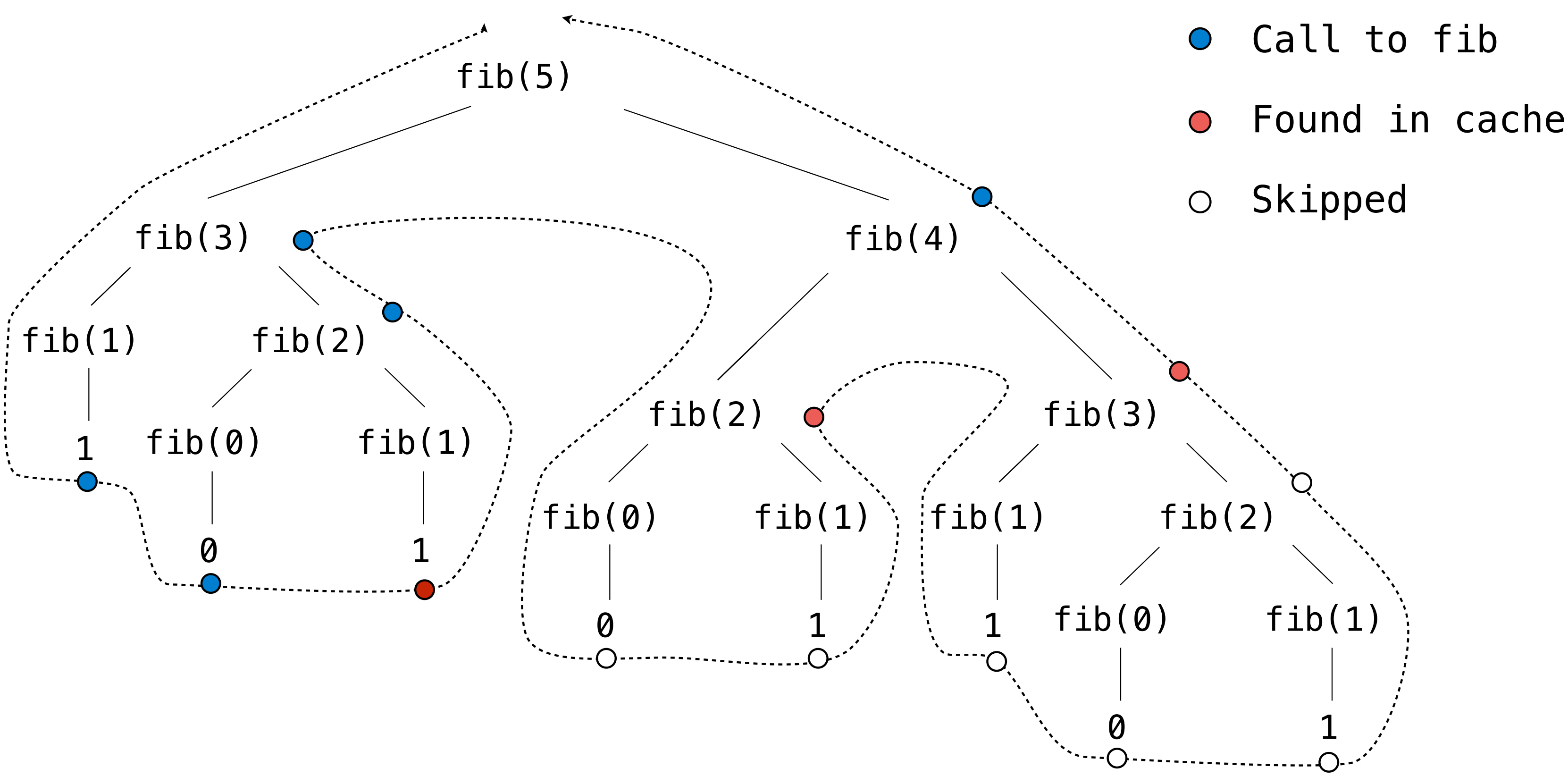

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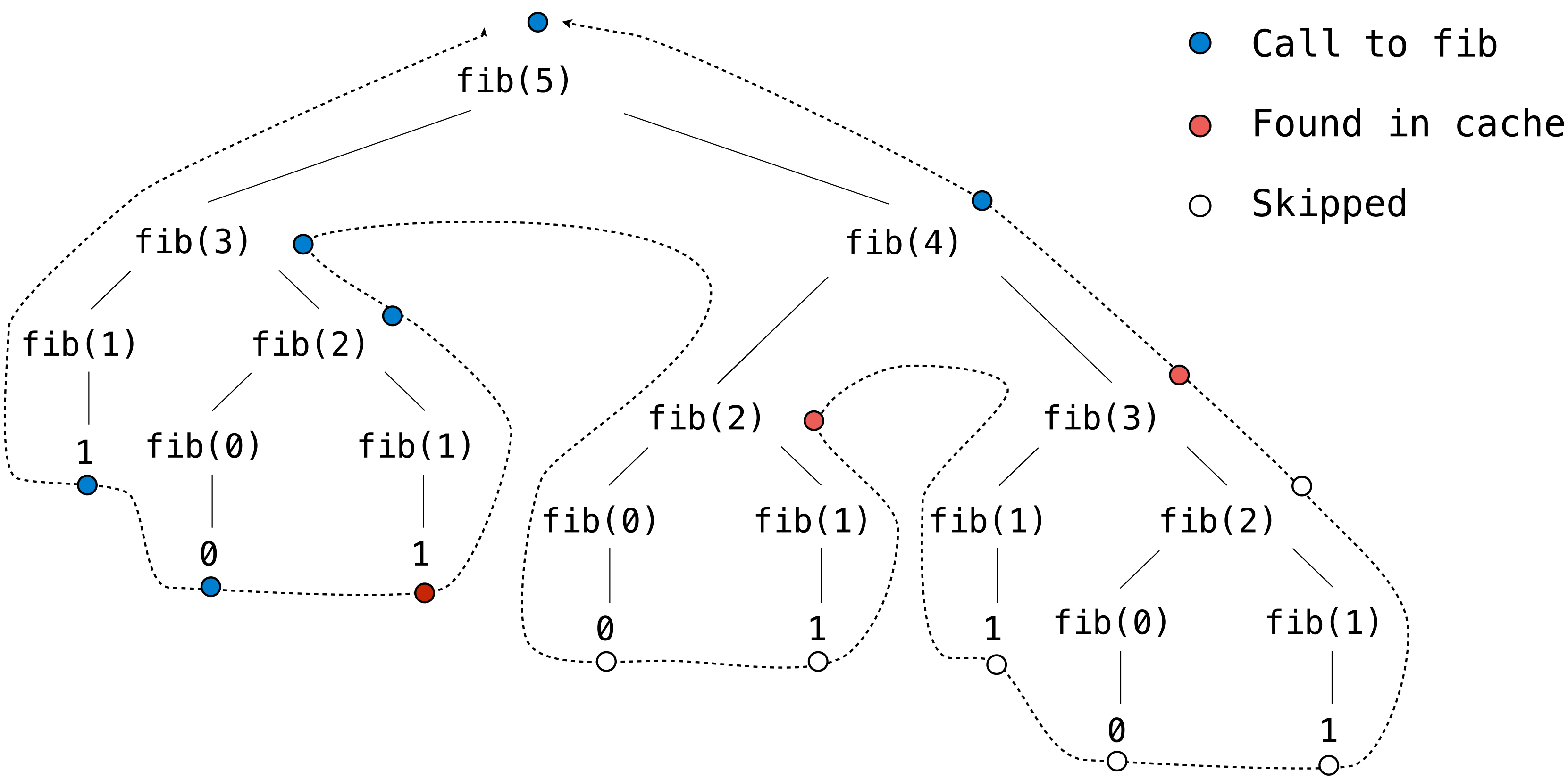

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        for branch in branches:
            assert isinstance(branch, Tree)
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        for branch in branches:
            assert isinstance(branch, Tree)
```

Built-in isinstance function: returns True if branch has a class that *is* **or** *inherits from* Tree

Tree Class

A Tree has an entry (any value) at its root and a list of branches

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class Tree:
    def __init__(self, entry, branches=()):
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def fib_tree(n):
    if n == 0 or n == 1:
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(Demo)

Hailstone Trees

Hailstone Trees

Hailstone Trees

`Pick a positive integer n as the start`

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

Hailstone Trees

Pick a positive integer n as the start

If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

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Continue this process until n is 1

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1

2

4

Hailstone Trees

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Pick a positive integer n as the start

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1

2

4

8

Hailstone Trees

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1

2

4

8

16

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1

2

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8

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64

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1

2

4

8

16

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64

128

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 1
 |
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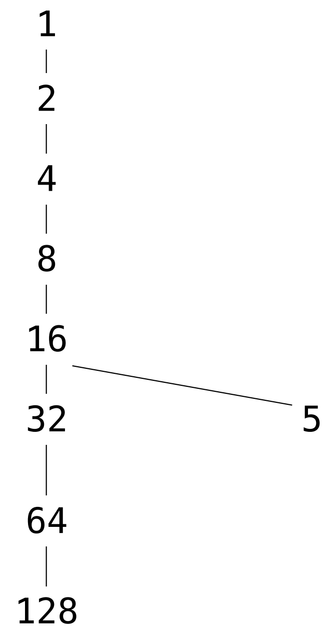

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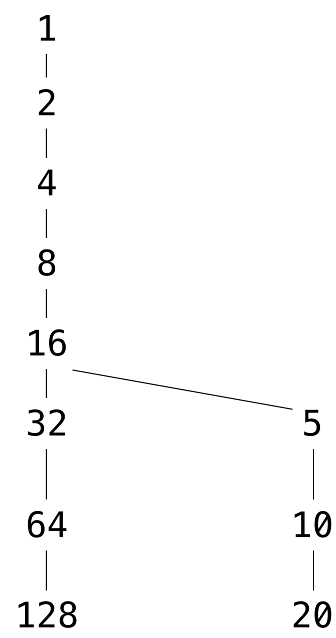

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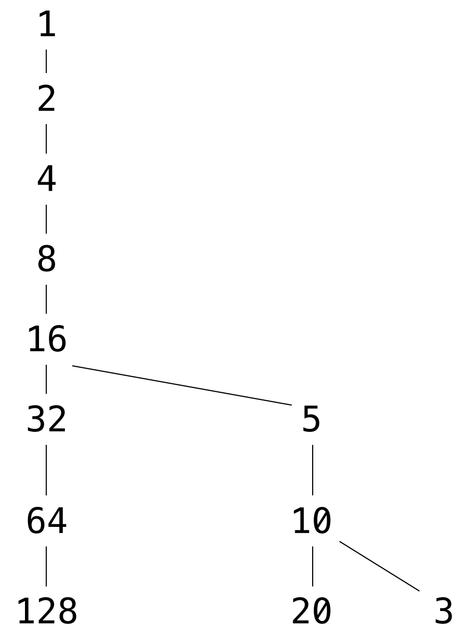

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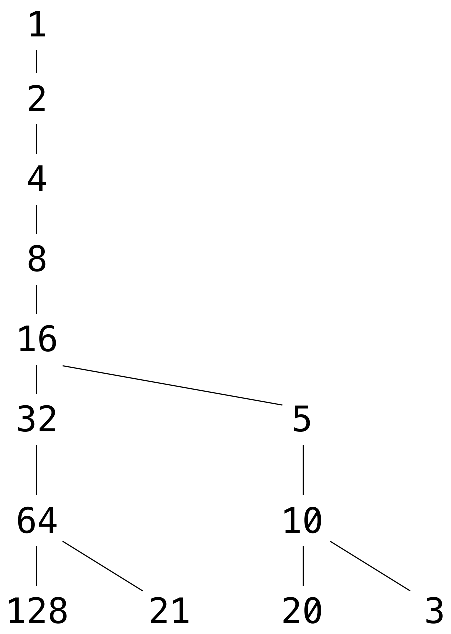

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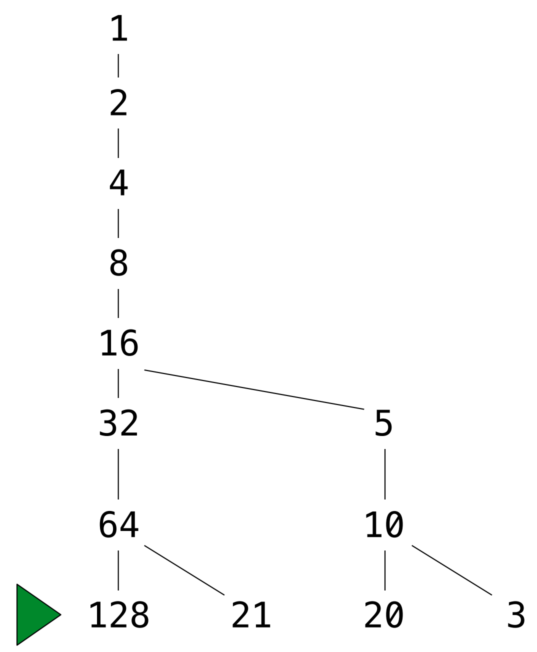

All possible n that start a
length-8 hailstone sequence

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```python
def hailstone_tree(k, n=1):
    """Return a Tree in which the paths from the
    leaves to the root are all possible hailstone
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```
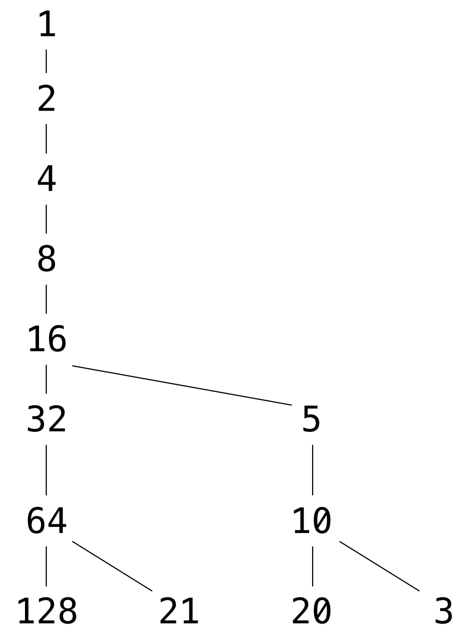

All possible n that start a length-8 hailstone sequence



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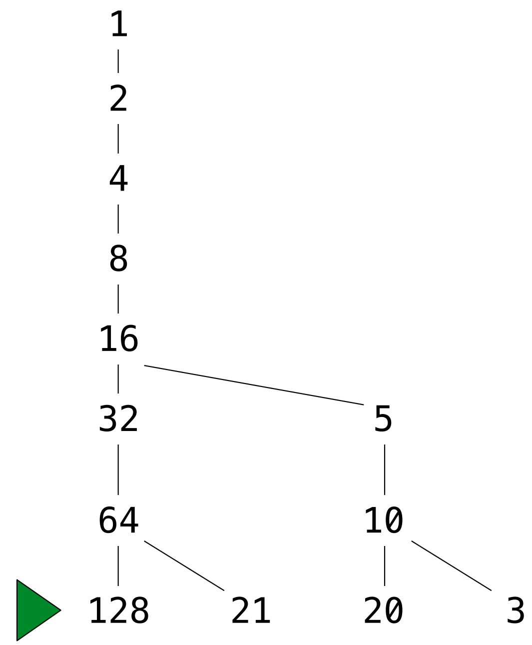

(Demo)

Binary Tree Class

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A binary tree is a tree that has
a left branch and a right branch
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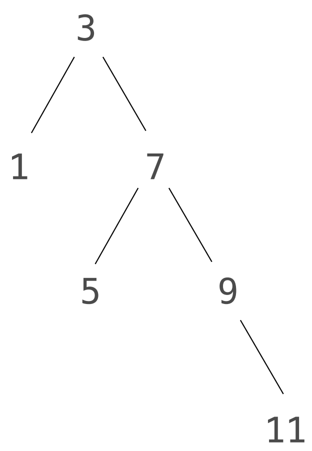

Binary Tree Class

A binary tree is a tree that has
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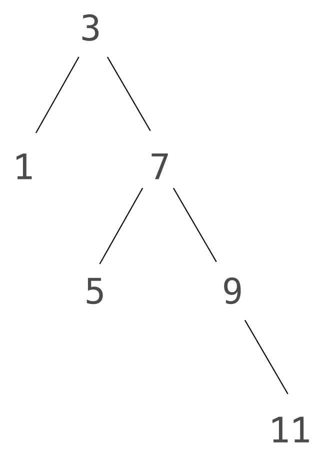
Idea: Fill the place of a missing
left branch with an empty tree



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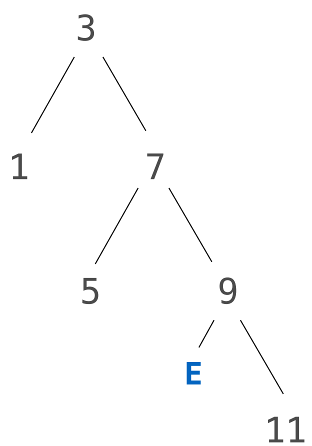

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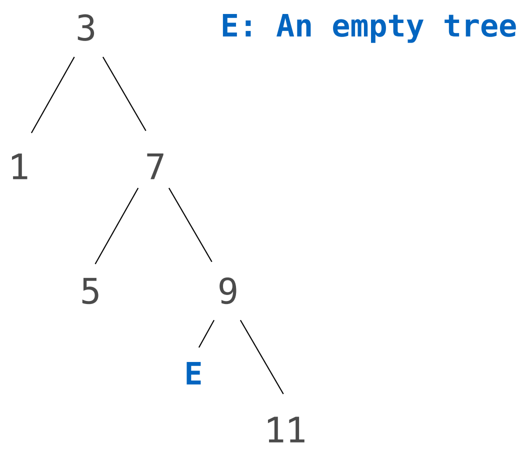

Binary Tree Class

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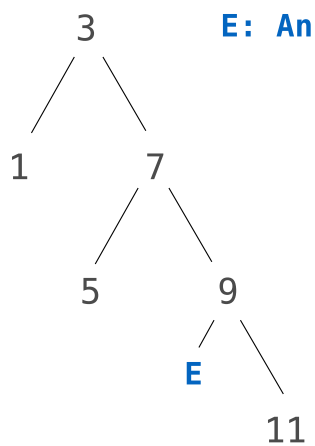
Idea: Fill the place of a missing left branch with an empty tree

Idea 2: An instance of BinaryTree always has *exactly* two branches



E: An empty tree

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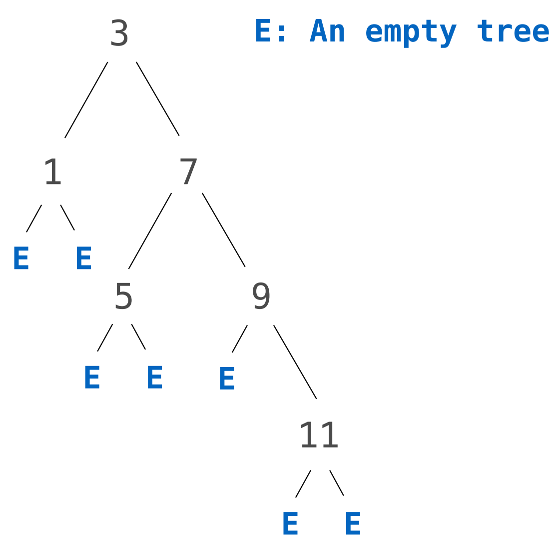

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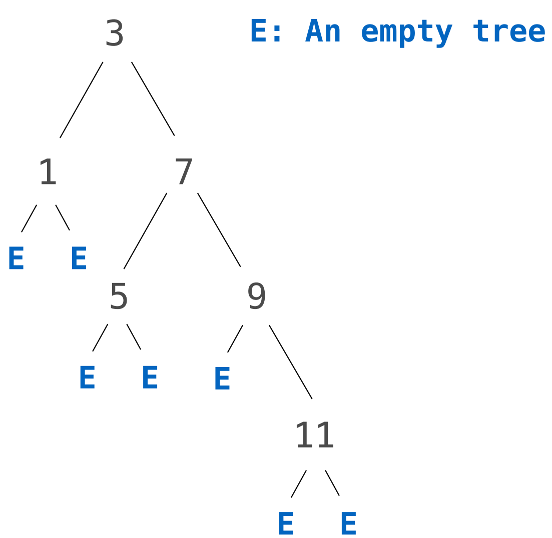


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class BinaryTree(Tree):
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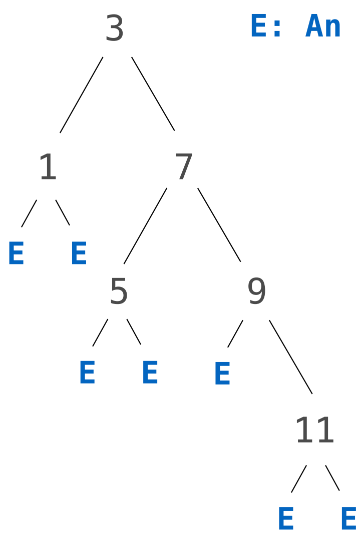

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```
class BinaryTree(Tree):
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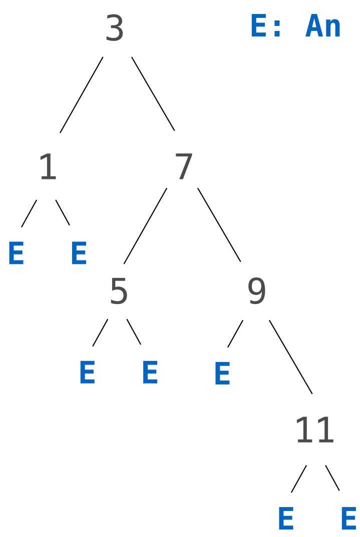

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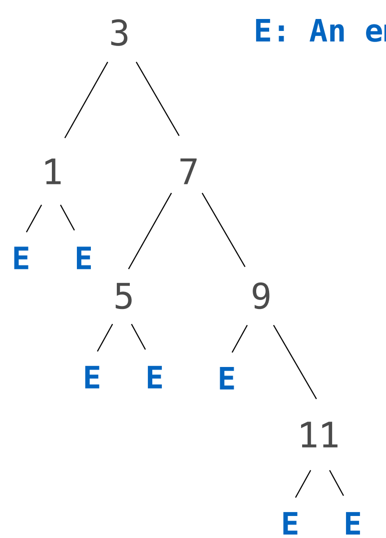

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    @property
    def left(self):
        return self.branches[0]
```

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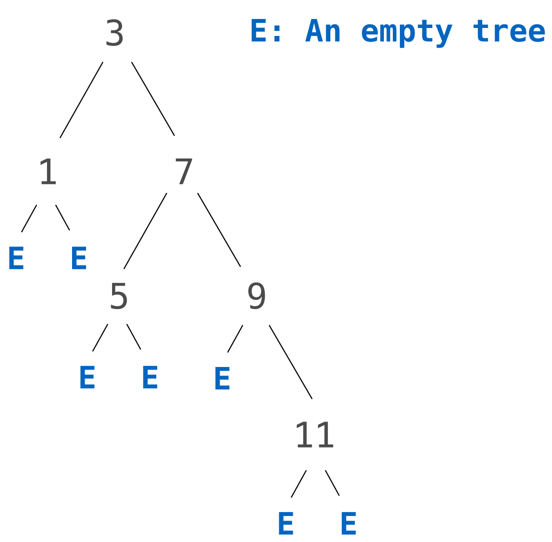


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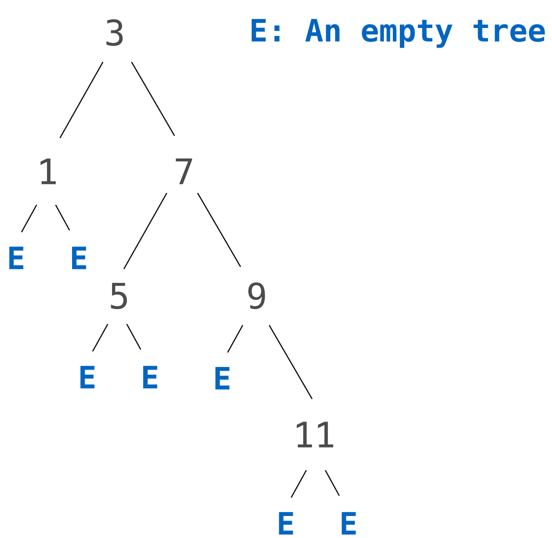


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Bin = BinaryTree
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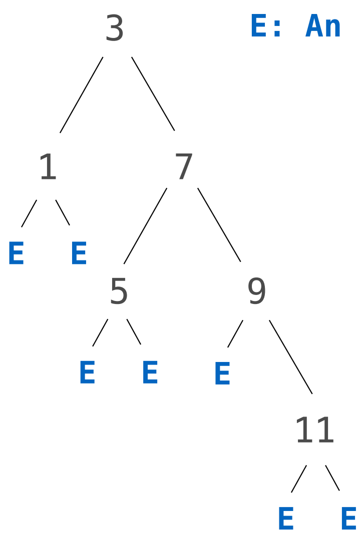

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Bin = BinaryTree
t = Bin(3, Bin(1),
           Bin(7, Bin(5),
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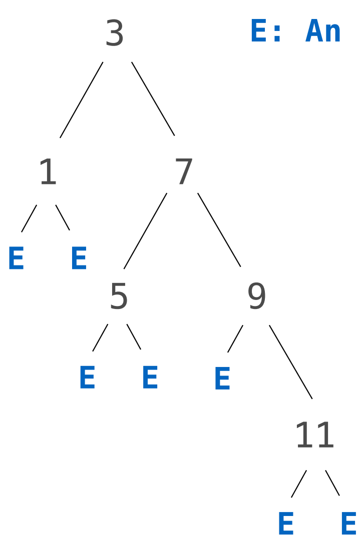


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