61A Lecture 19

Monday, March 9

Announcements

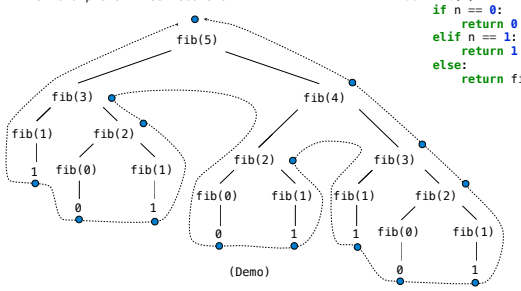
- Project 3 due Thursday 3/12 @ 11:59pm
  - Project party on Tuesday 3/10 5pm–6:30pm in 2050 VLSB
  - Bonus point for early submission by Wednesday 3/11
- Guerrilla section this weekend (details announced soon)
- Homework 6 due Monday 3/16 @ 11:59pm
- Midterm 2 is on Thursday 3/19 7pm–9pm
  - Fill out conflict form if you cannot attend due to a course conflict

Measuring Efficiency

Recursive Computation of the Fibonacci Sequence

Our first example of tree recursion:

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-2) + fib(n-1)
```



(Demo)

http://en.wikipedia.org/wiki/File:Fibonacci.jpg

Memoization

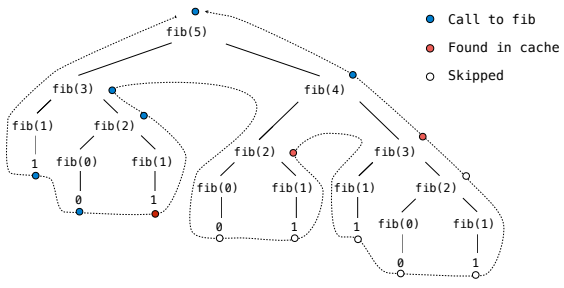
Memoization

Idea: Remember the results that have been computed before

```
def memo(f):
    cache = {}                          # Keys are arguments that map to return values
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized                     # Same behavior as f, if f is a pure function
```

(Demo)

Memoized Tree Recursion



- Call to fib
- Found in cache
- Skipped

Tree Class

Tree Class

A Tree has an entry (any value) at its root and a list of branches

```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)


def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        return Tree(left.entry + right.entry, (left, right))
```

Built-in isinstance function: returns True if branch has a class that *is* or *inherits from* Tree

(Demo)

Hailstone Trees

Hailstone Trees

Hailstone Trees

Pick a positive integer n as the start

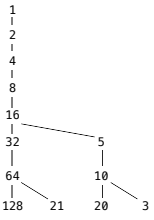
If n is even, divide it by 2

If n is odd, multiply it by 3 and add 1

Continue this process until n is 1

```
def hailstone_tree(k, n=1):
    """Return a Tree in which the paths from the
    leaves to the root are all possible hailstone
    sequences of length k ending in n."""
```

All possible n that start a length-8 hailstone sequence



(Demo)

Binary Tree Class

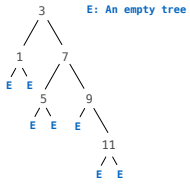
Binary Tree Class

Binary Tree Class

A binary tree is a tree that has a left branch and a right branch

Idea: Fill the place of a missing left branch with an empty tree

Idea 2: An instance of BinaryTree always has *exactly* two branches




```
class BinaryTree(Tree):
    empty = Tree(None)
    empty.is_empty = True

    def __init__(self, entry, left=empty, right=empty):
        Tree.__init__(self, entry, (left, right))
        self.is_empty = False

    @property
    def left(self):
        return self.branches[0]

    @property
    def right(self):
        return self.branches[1]

Bin = BinaryTree
t = Bin(3, Bin(1),
           Bin(7, Bin(5),
                  Bin(9, Bin.empty,
                         Bin(11))))
```

(Demo)