61A Extra Lecture 1

Thursday, January 29

Announcements

- If you want 1 unit (pass/no pass) of credit for this CS 98, you need to:
  - Enroll in "Additional Topics on the Structure and Interpretation of Computer Programs"
  - Course control number: 25709
  - Concurrently enroll in CS 61A
  - Complete ~6 difficult assignments, which may be released/due at strange times
  - Only for people who really want extra work that's beyond the scope of normal CS 61A
- Anyone is welcome to attend the extra lectures, whether or not they enroll
- Lectures will be on Thursdays 5–6:30 PM in 2050 VLSB; A schedule will be posted eventually
- John's office hours: 10am–12pm Wednesday & Friday by appt. (denero.org/meet) in 781 Soda

Lambda Expressions

(Demo)

Lambda Expressions

```
>>> x = 10
>>> square = x * x
>>> square = lambda x: x * x
>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function with formal parameter x that returns the value of "x * x"

Must be a single expression

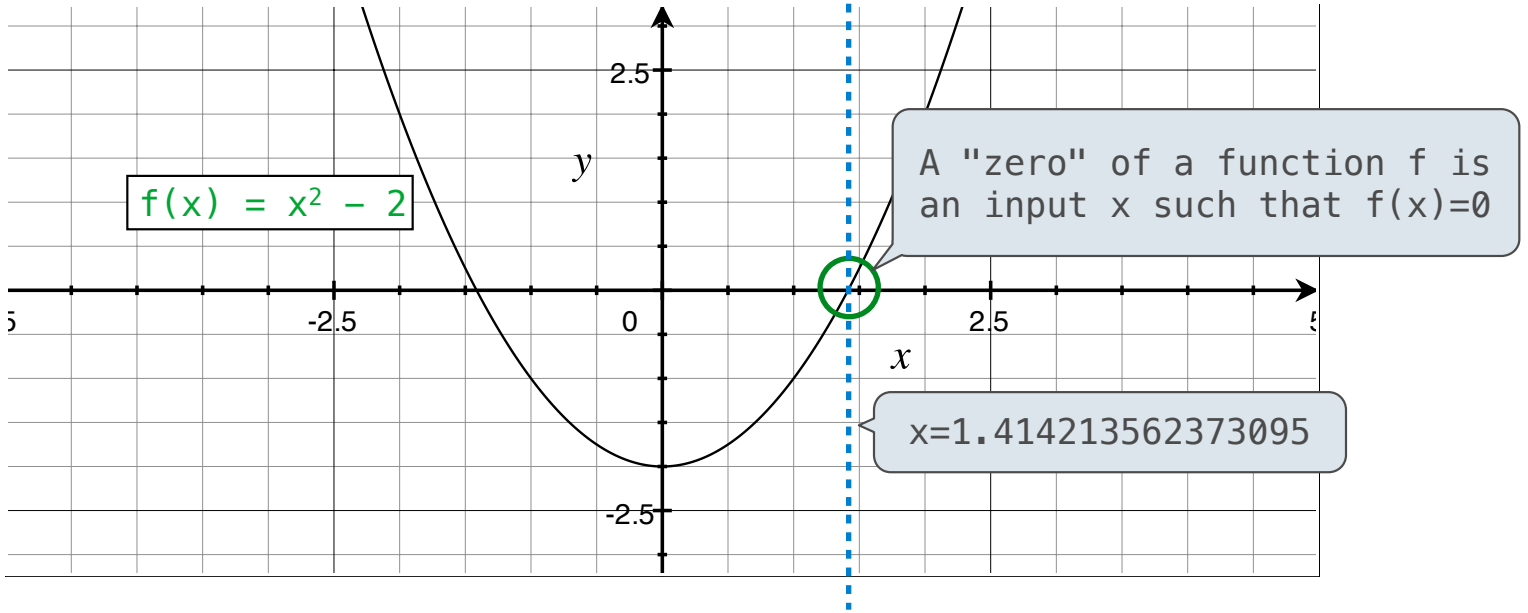
Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

Newton's Method

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable functions!



Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is $\sqrt{a}$. (We're solving the equation $x^2 = a$.)

Newton's Method

Given a function f and initial guess x,

Repeatedly improve x:

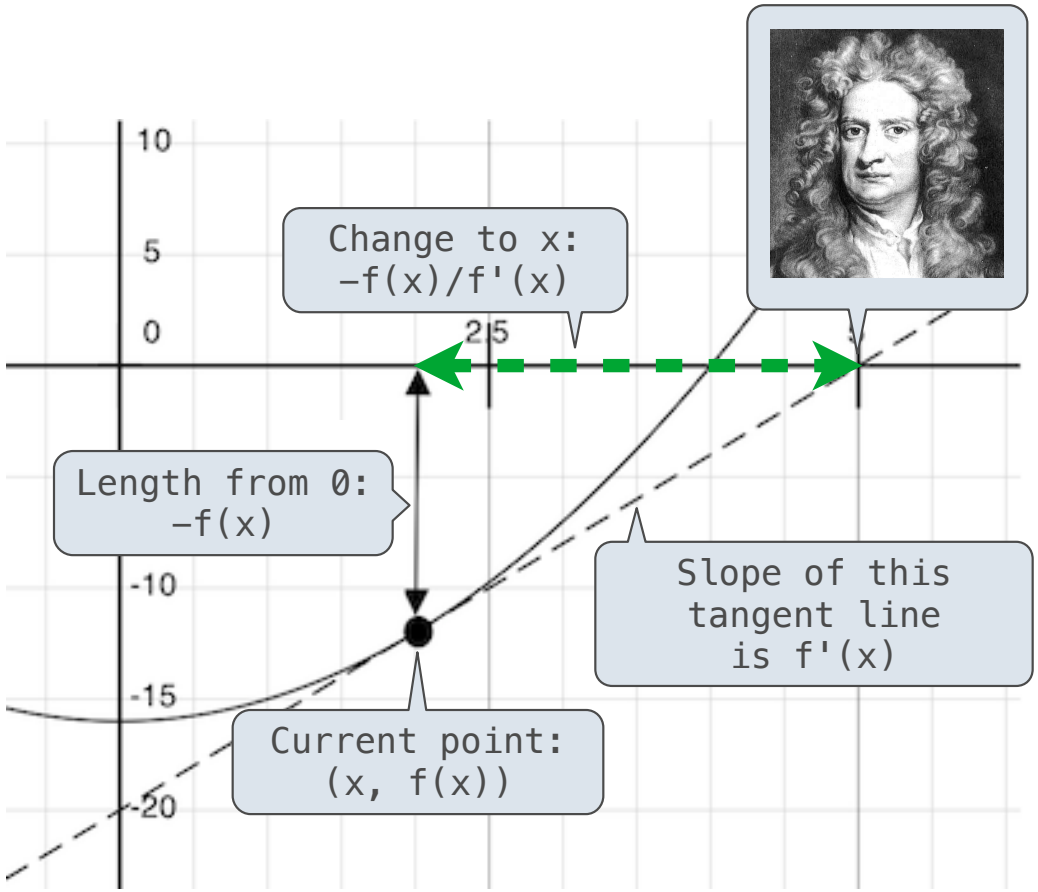
Compute the value of f
at the guess: f(x)

Compute the derivative
of f at the guess: f'(x)

Update guess x to be:

$$x - \frac{f(x)}{f'(x)}$$

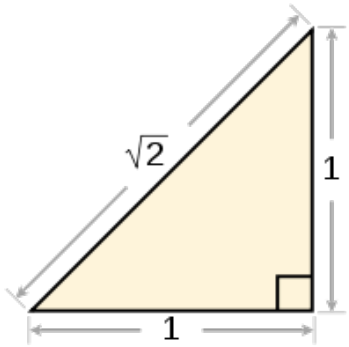
Finish when f(x) = 0 (or close enough)



http://en.wikipedia.org/wiki/File:NewtonIteration_Ani.gif

Using Newton's Method

How to find the square root of 2?

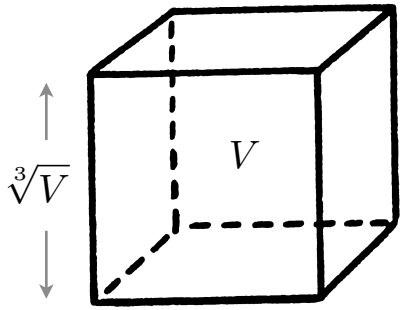



```
>>> f  = lambda x: x*x - 2
>>> df = lambda x: 2*x
>>> find_zero(f, df)
1.4142135623730951
```

$f(x) = x^2 - 2$
$f'(x) = 2x$

Applies Newton's method

How to find the cube root of 729?




```
>>> g  = lambda x: x*x*x - 729
>>> dg = lambda x: 3*x*x
>>> find_zero(g, dg)
9.0
```

$g(x) = x^3 - 729$
$g'(x) = 3x^2$

Iterative Improvement

Special Case: Square Roots

How to compute square_root(a)

Idea: Iteratively refine a guess x about the square root of a

Update: $$x = \frac{x + \frac{a}{x}}{2}$$

Babylonian Method

(Demo)

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Special Case: Cube Roots

How to compute cube_root(a)

Idea: Iteratively refine a guess x about the cube root of a

Update: $$x = \frac{2 \cdot x + \frac{a}{x^2}}{3}$$ (Demo)

Implementation questions:

What guess should start the computation?

How do we know when we are finished?

Implementing Newton's Method

(Demo)

Approximate Differentiation

Differentiation can be performed symbolically or numerically

f(x) = x^2 − 16

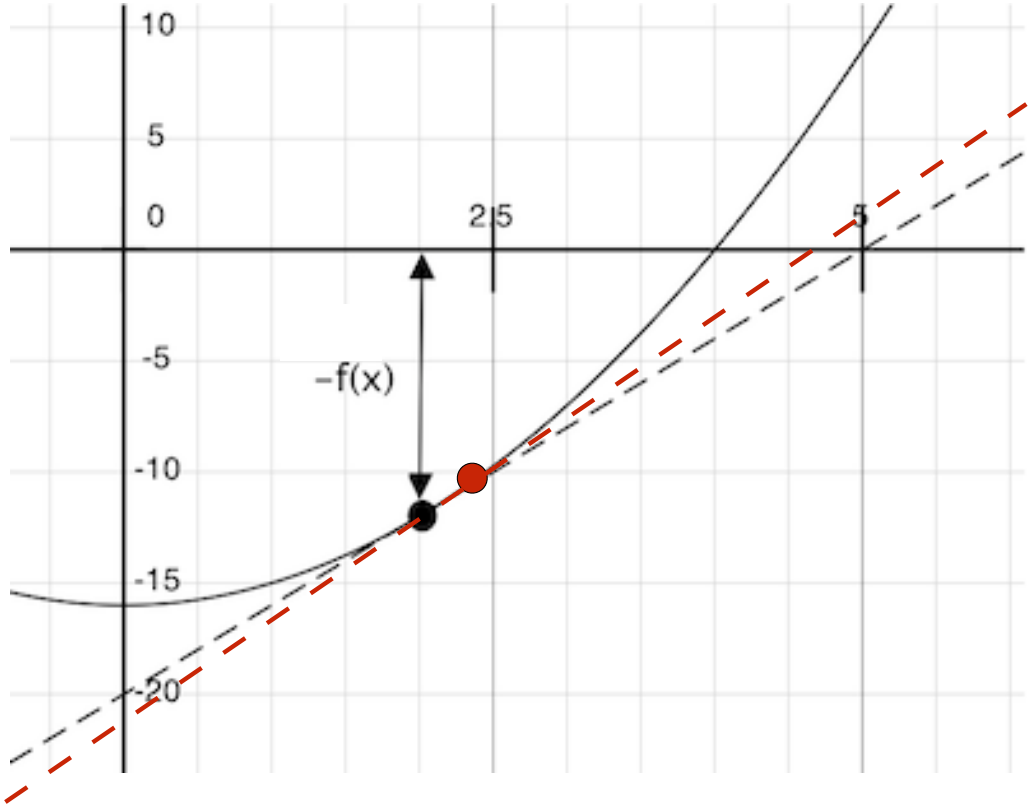
f'(x) = 2x

f'(2) = 4

$$f'(x) = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a}$$

$$f'(x) \approx \frac{f(x+a) - f(x)}{a}$$ (if a is small)

(Demo)



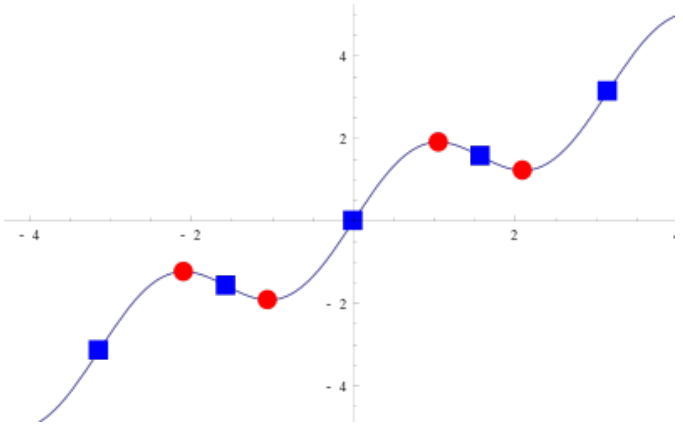
Critical Points and Inverses

Maxima, minima, and inflection points of a differentiable function occur when the derivative is 0

```
derive = lambda f: lambda x: slope(f, x)
```

The inverse $f^{-1}(y)$ of a differentiable, one-to-one function computes the value x such that f(x) = y

(Demo)



http://upload.wikimedia.org/wikipedia/commons/f/fd/Stationary_vs_inflection_pts.svg