61A Extra Lecture 2

Thursday, February 5

Announcements

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- If you want 1 unit (pass/no pass) of credit for this CS 98, you need to:

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- Extra Homework 1 due Thursday 2/12 @ 11:59pm

Dice

Hog: The End Game

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You: 98
Them: 99

Hog: The End Game

You: 98
Them: 99

You: 92
Them: 99

Hog: The End Game

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What is the chance that I'll score at least **k** points rolling **n** six-sided dice?

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S_n : Score from rolling n dice

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Base case: The chance to score at least 0 in 0 rolls is 1 (guaranteed)

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(Demo)

Memoization

Recursive Computation of the Fibonacci Sequence

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fib(5)

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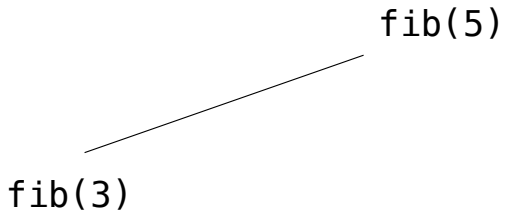


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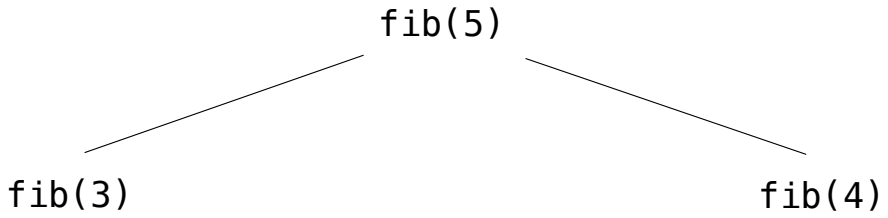


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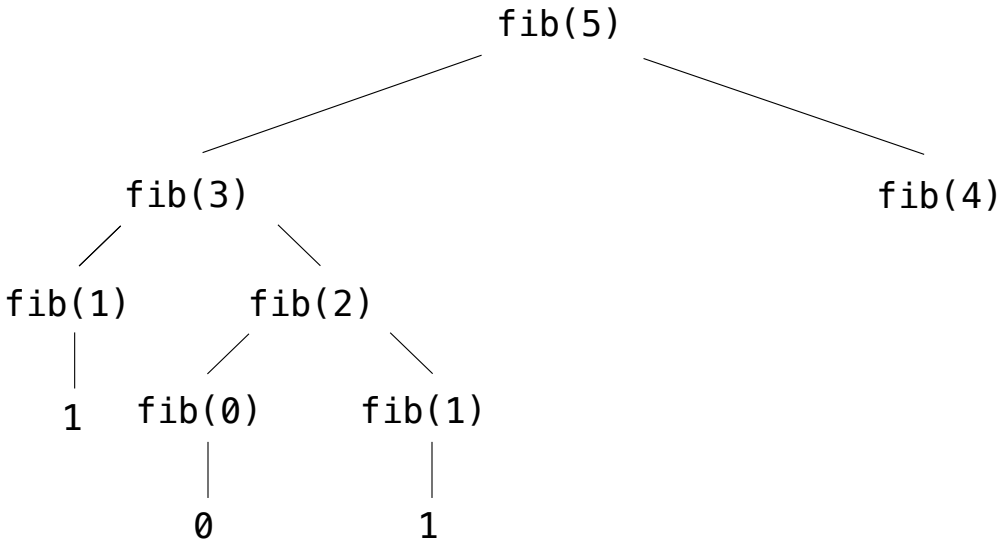


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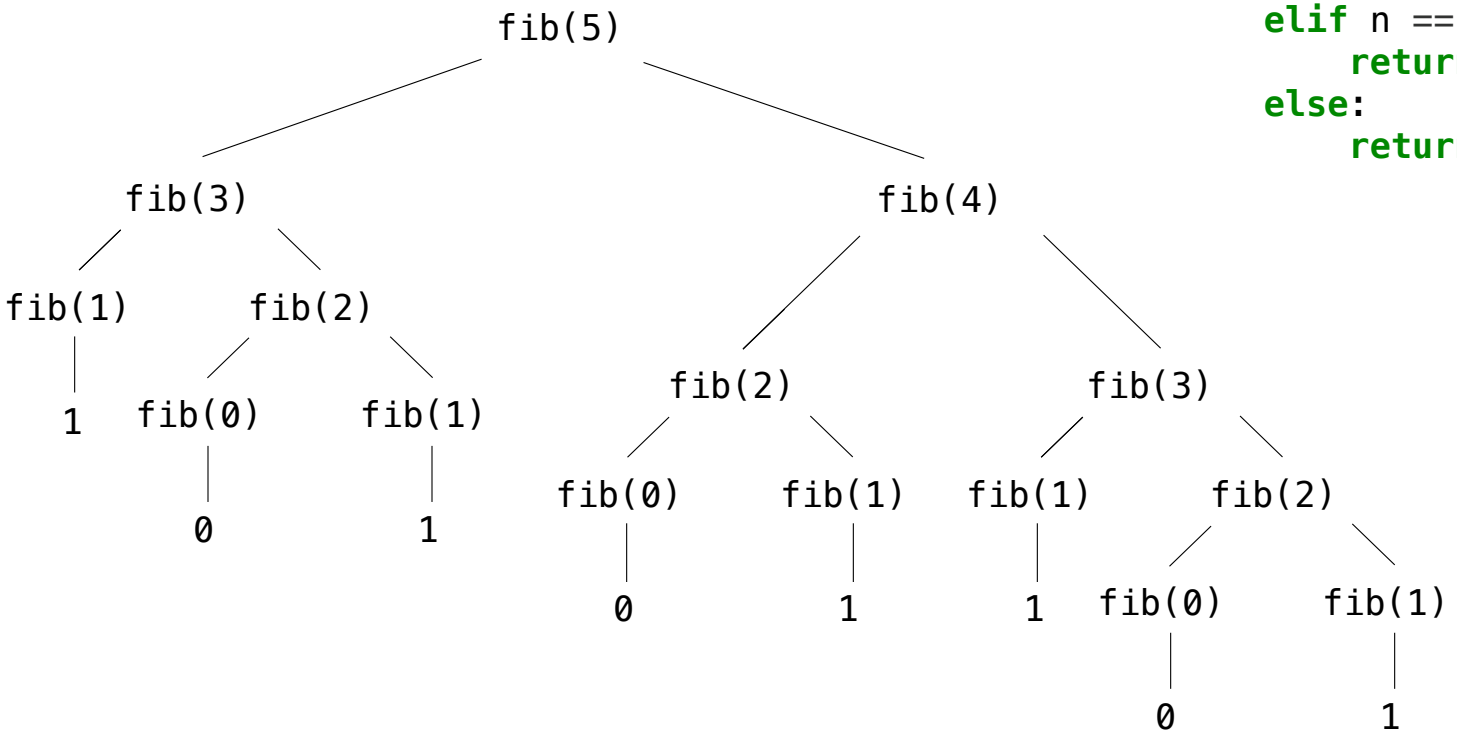

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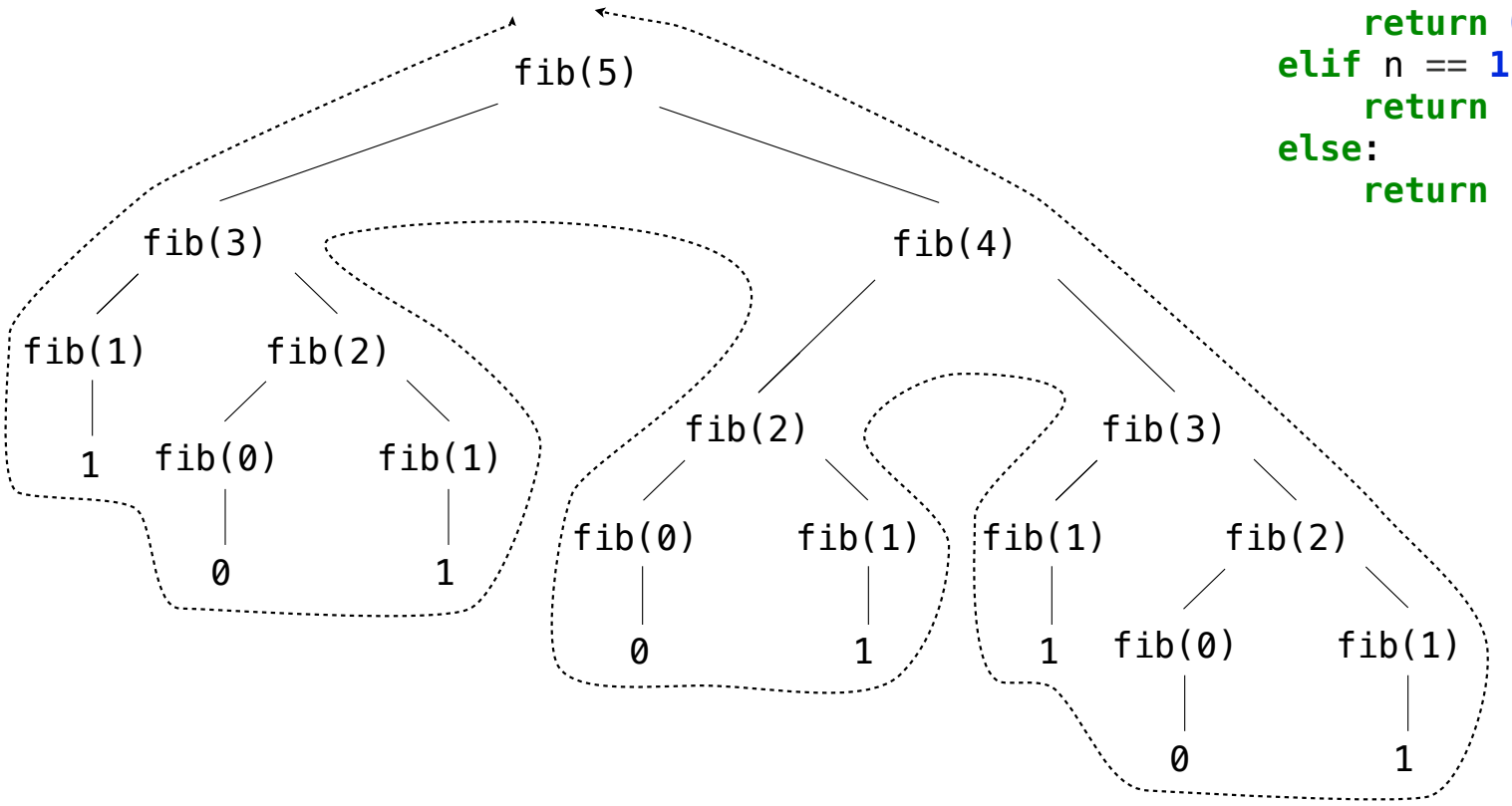


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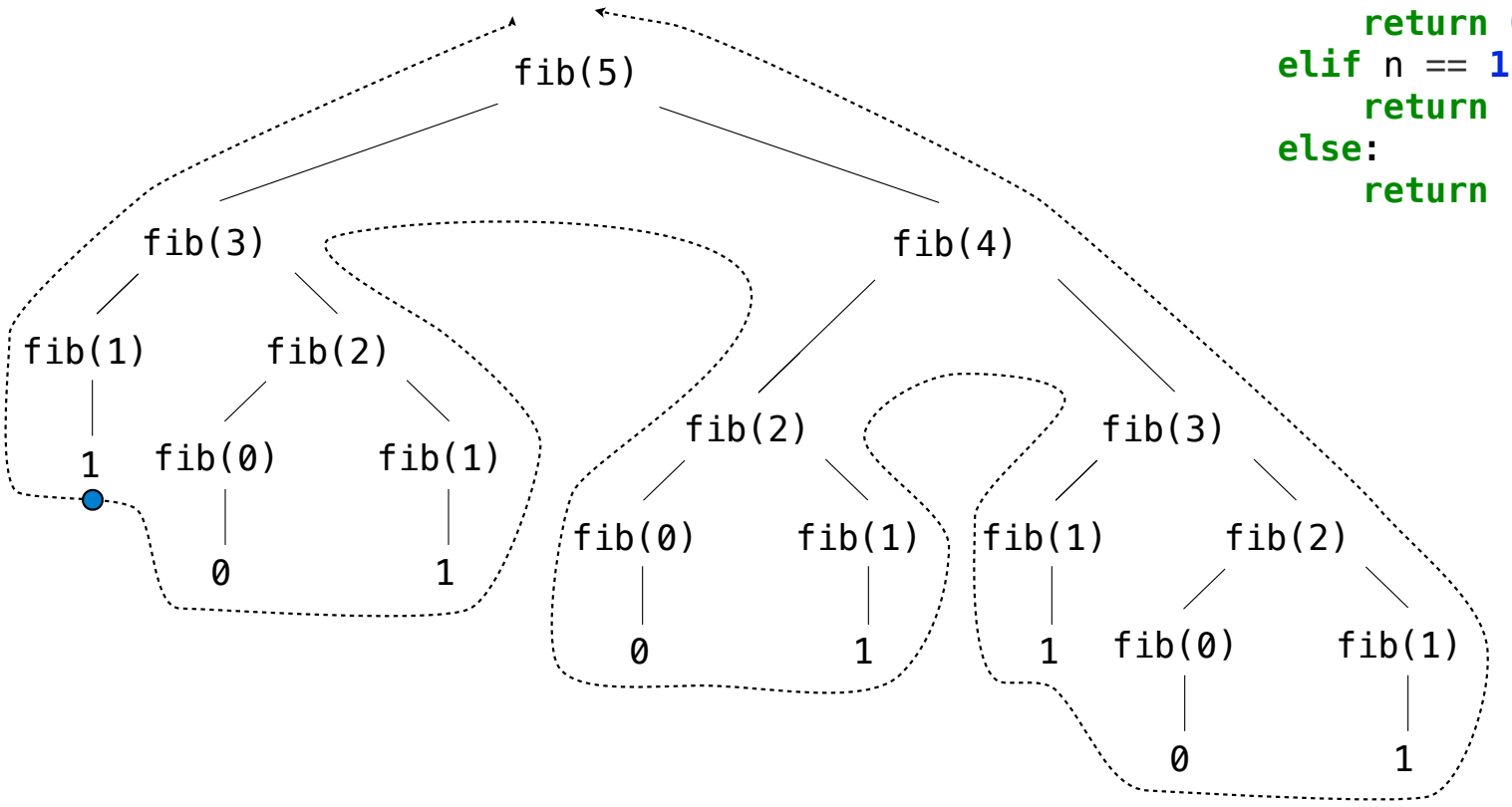

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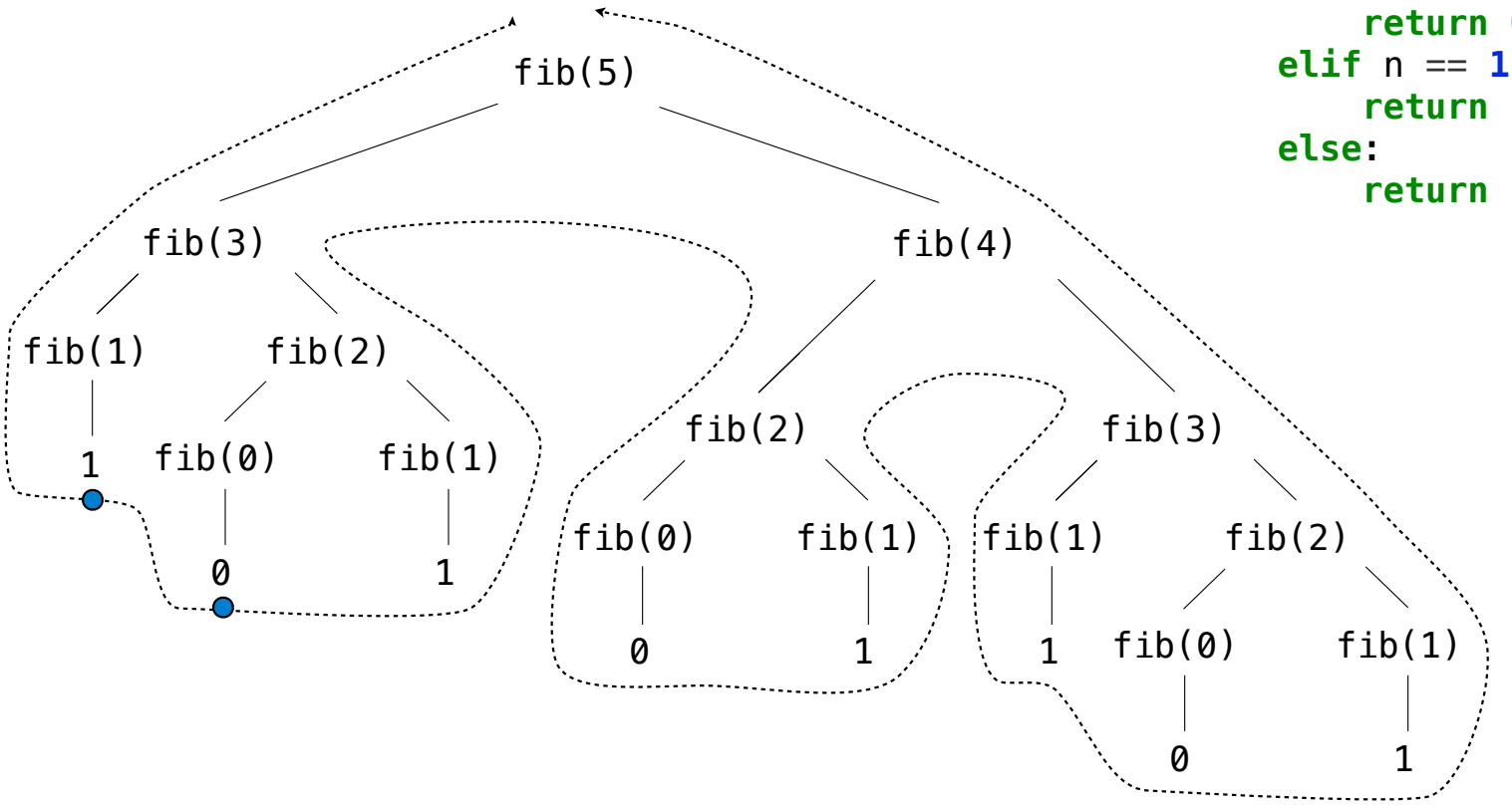


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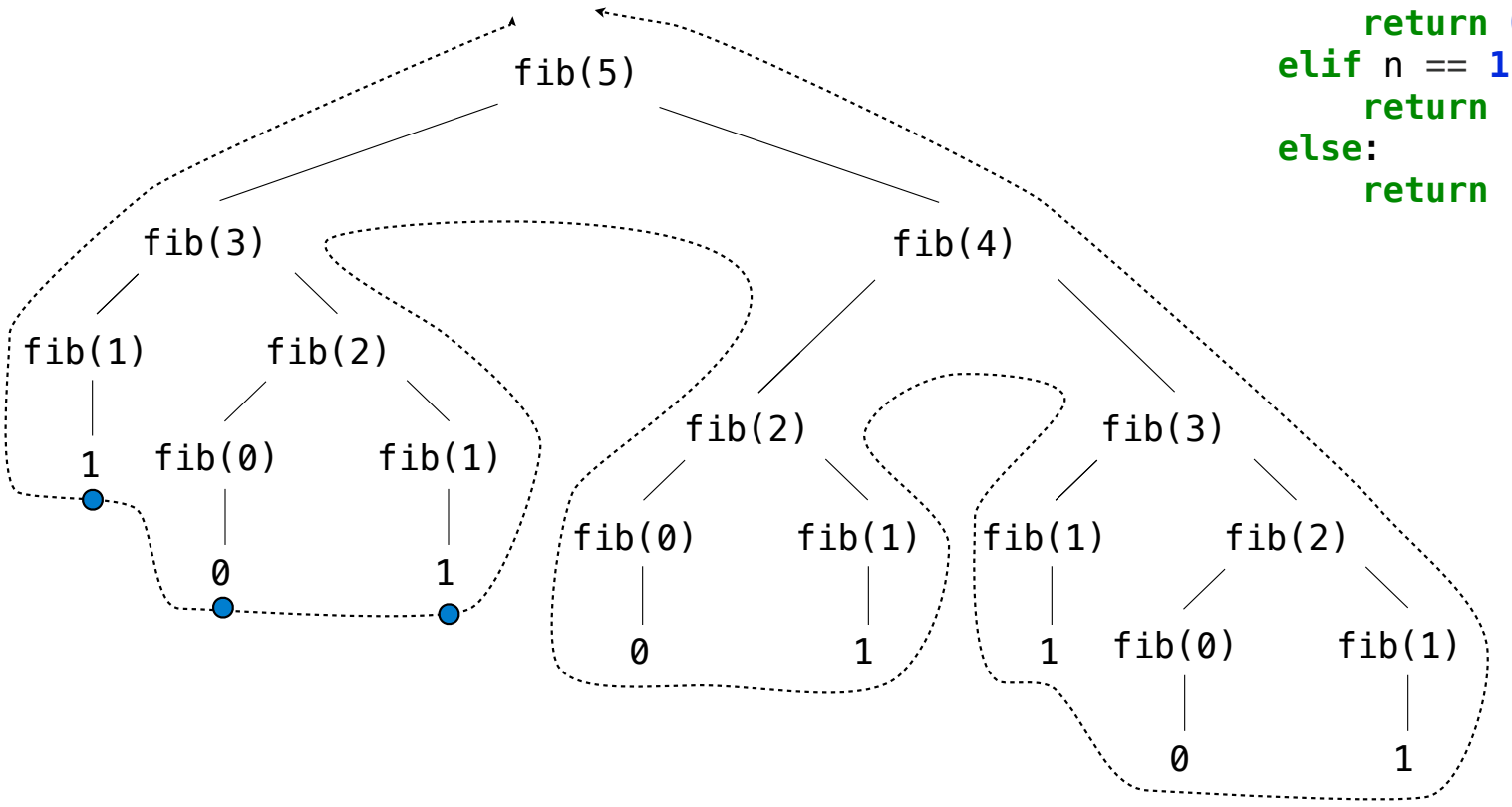

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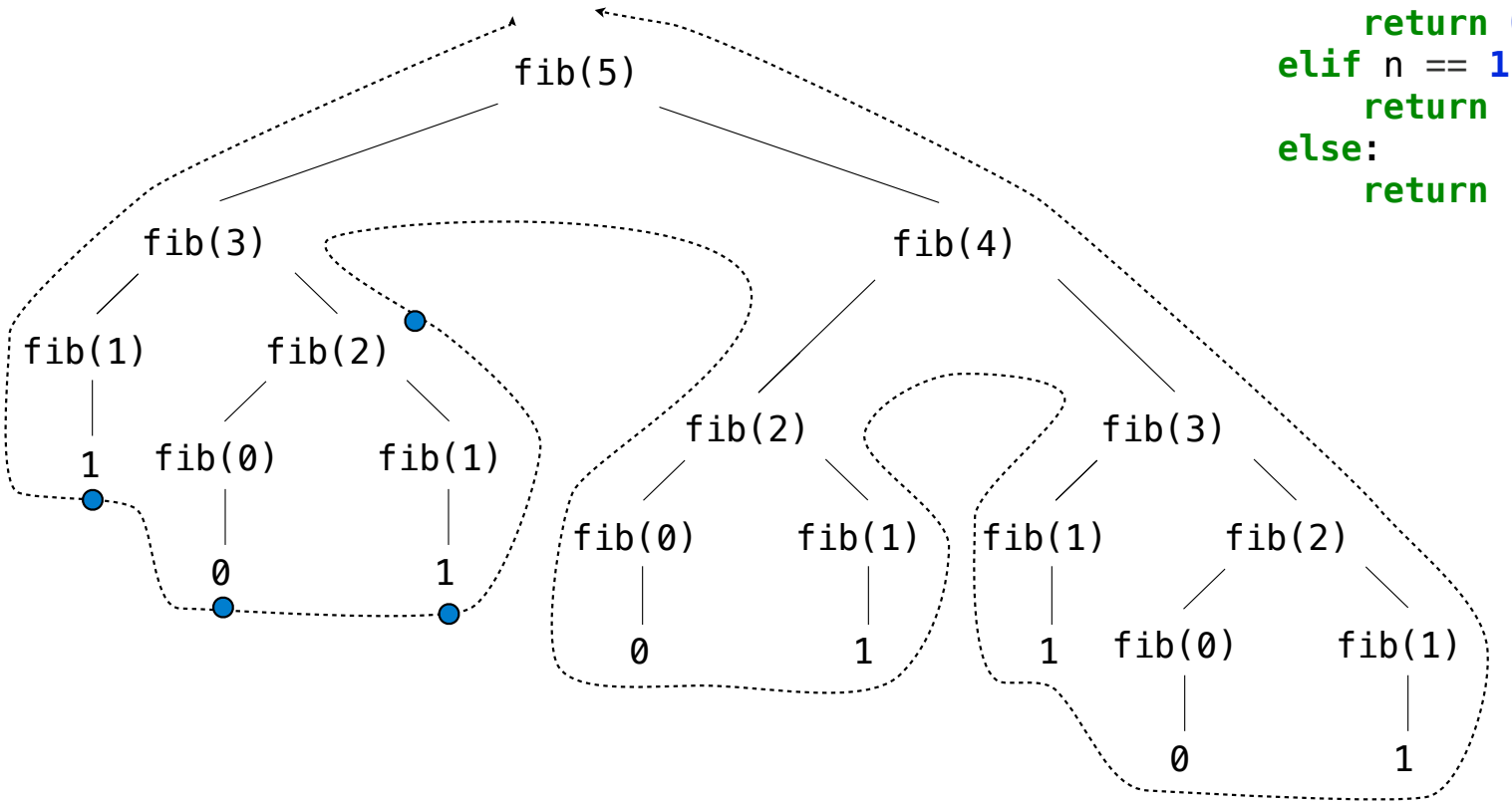

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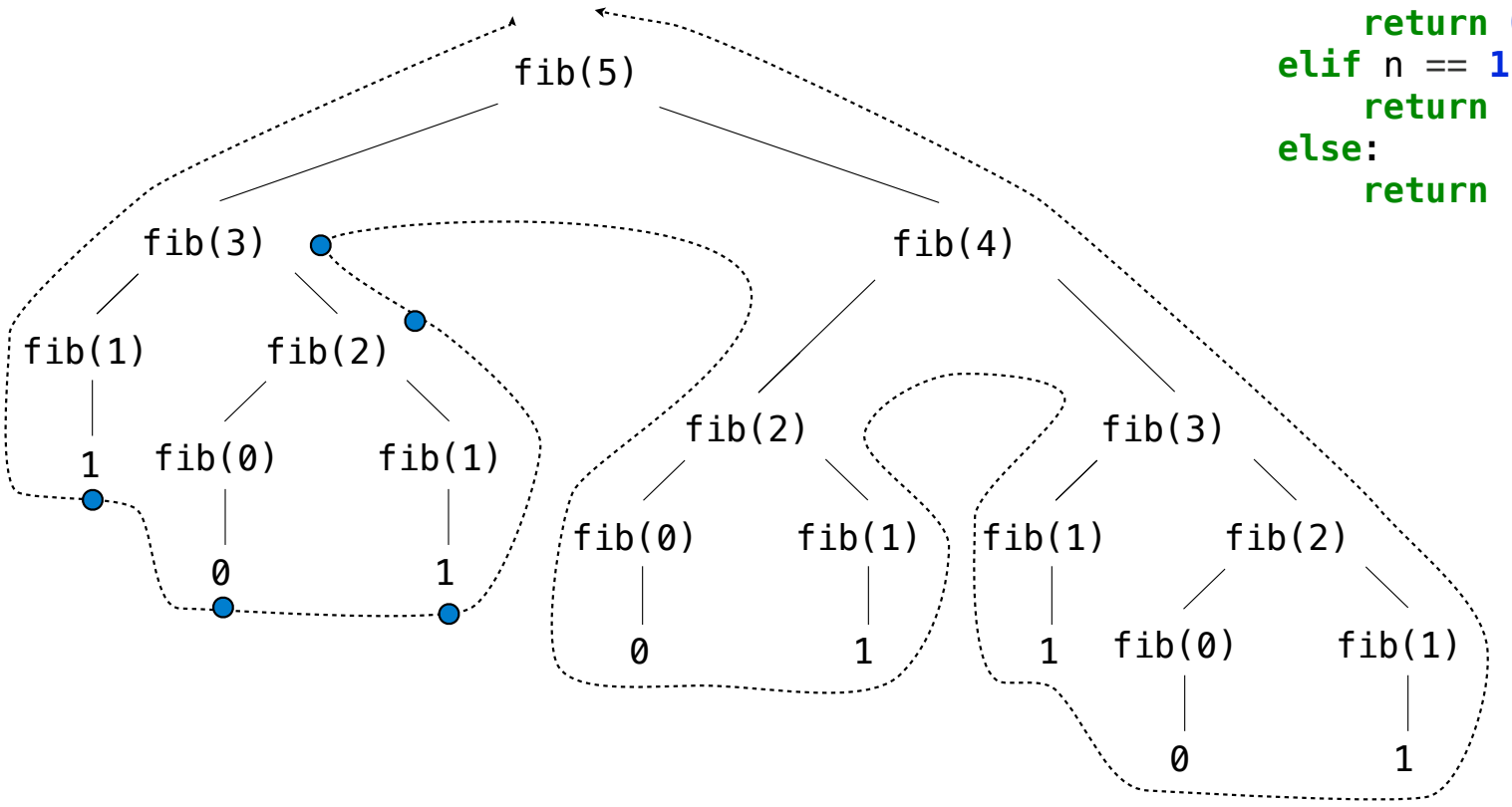

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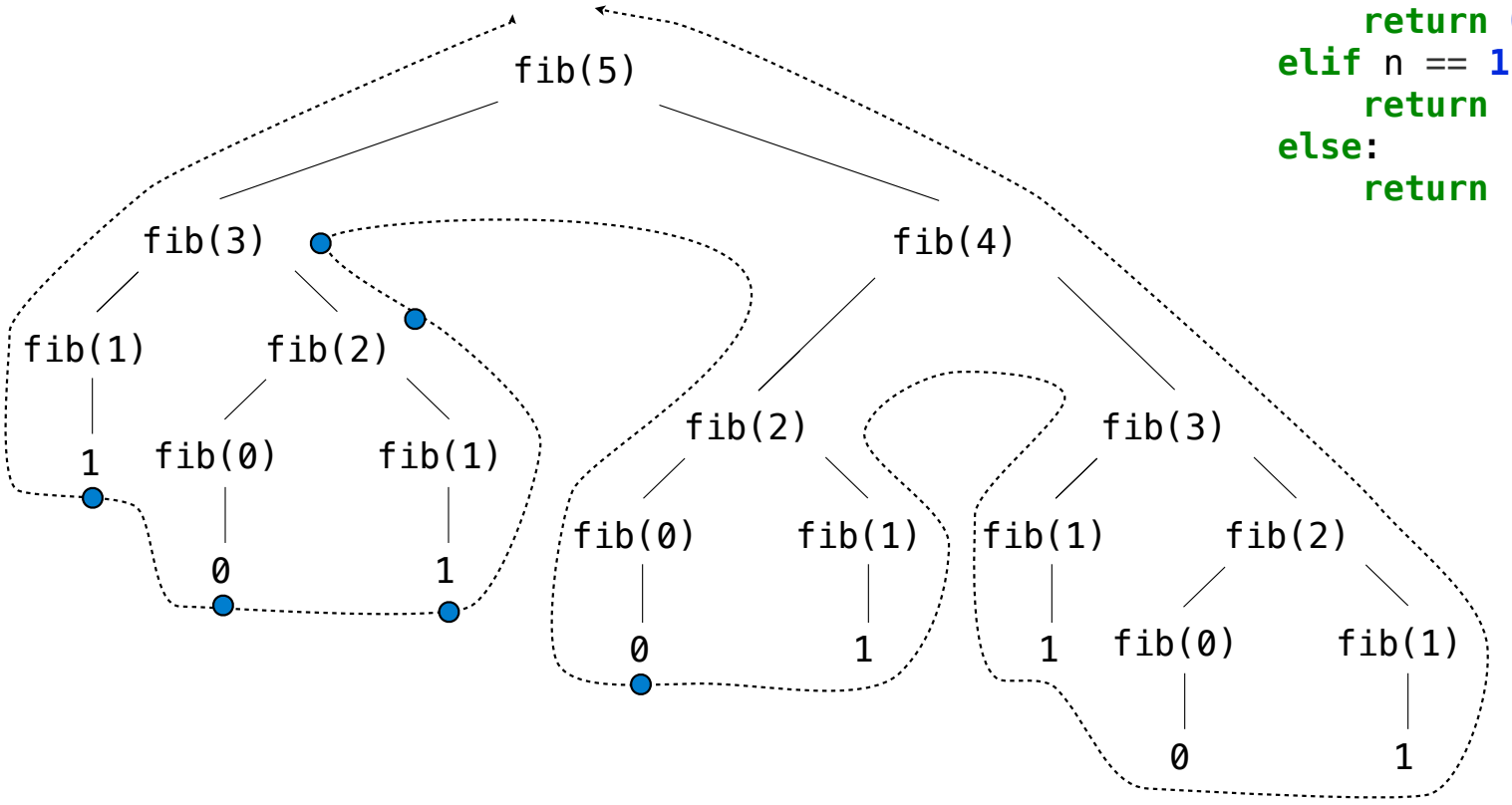

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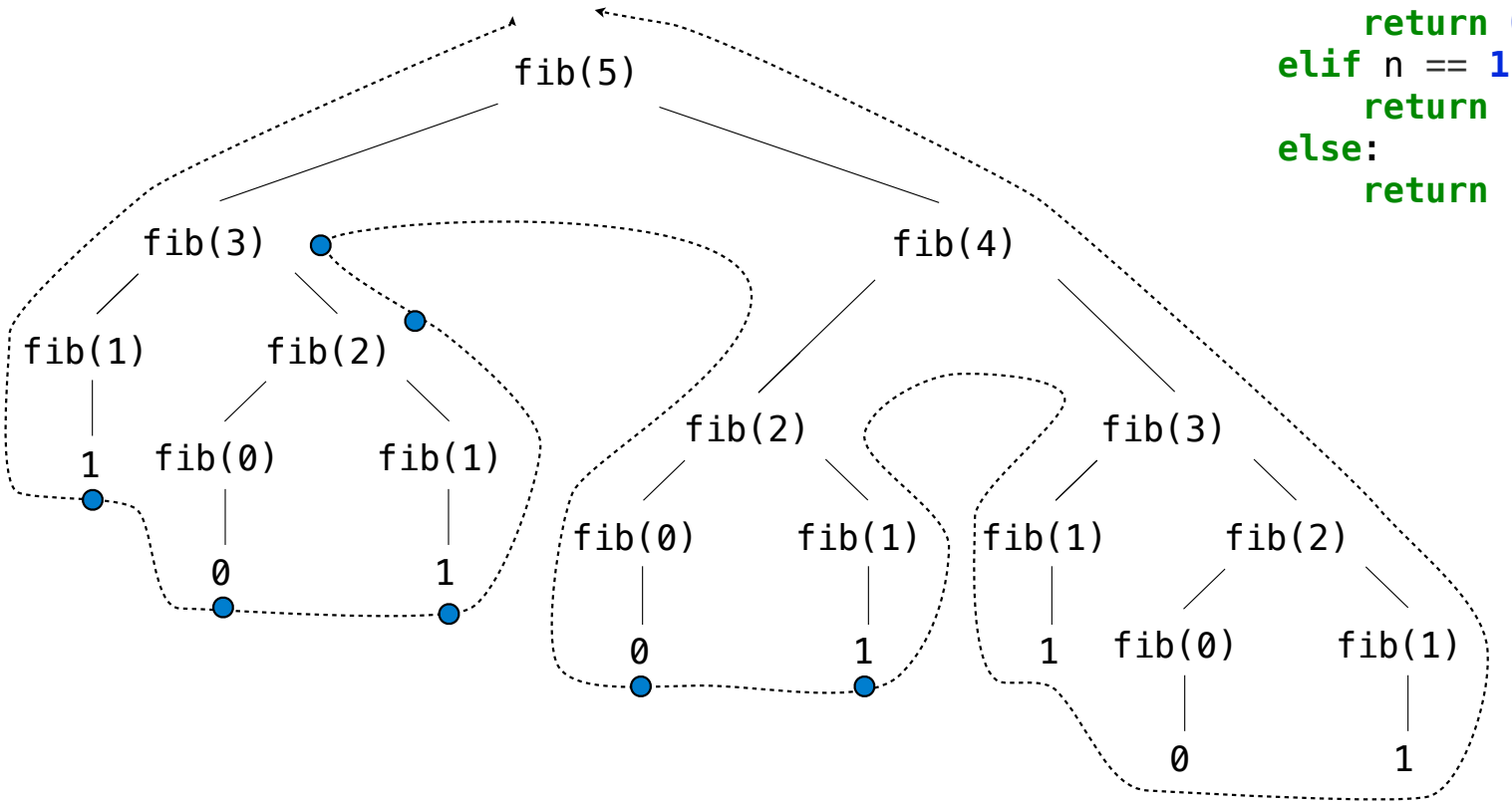

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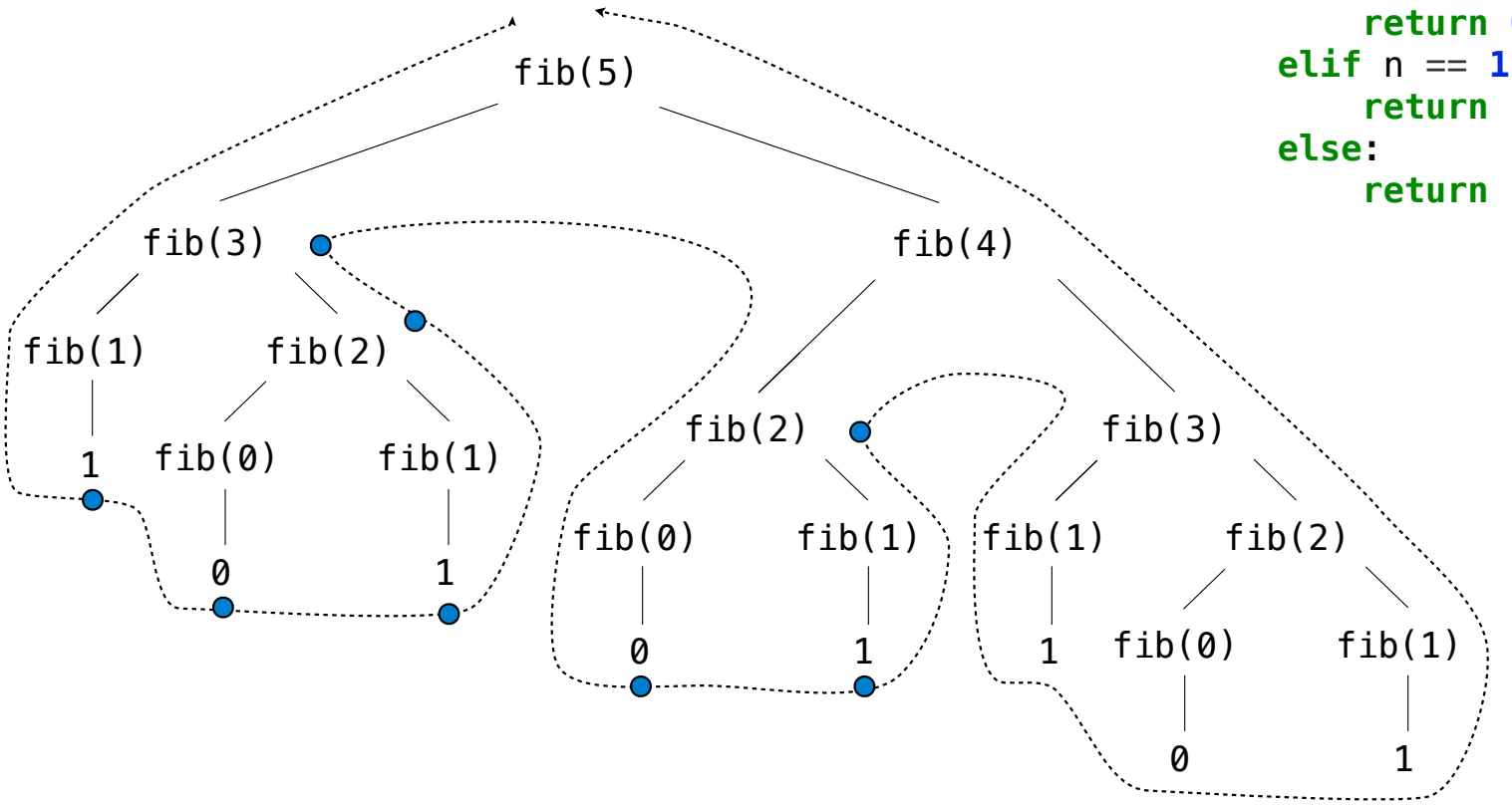

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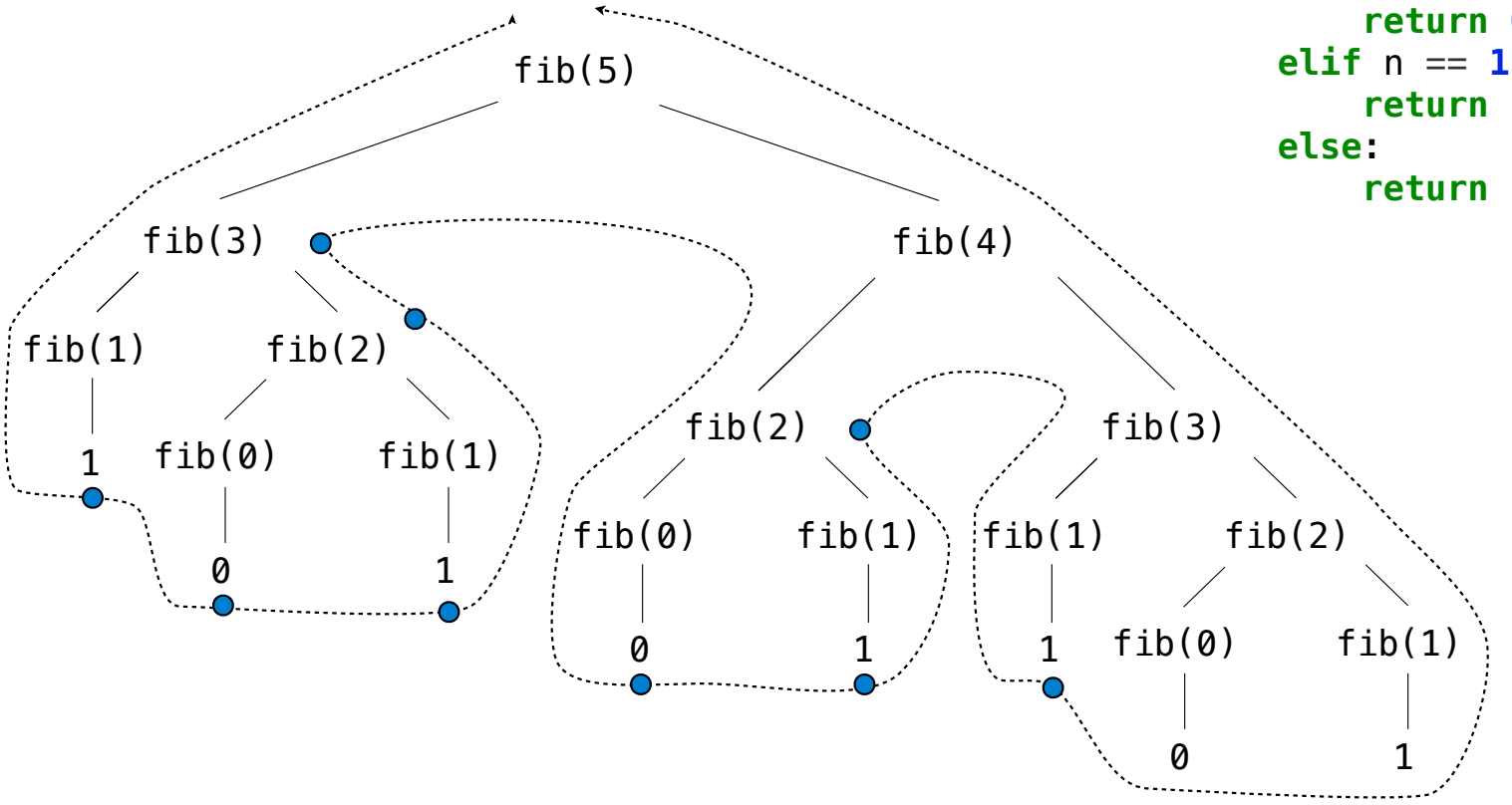

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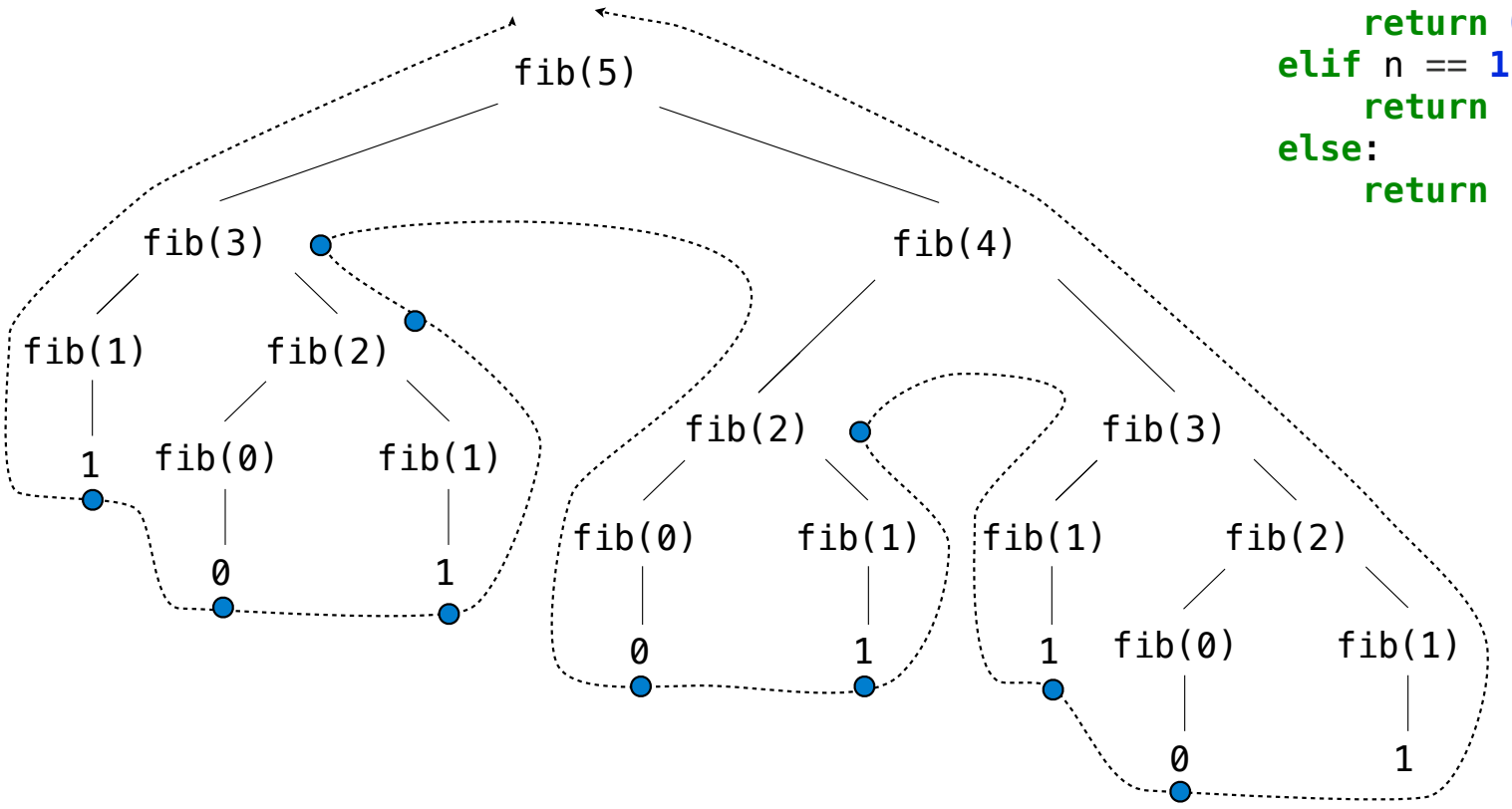

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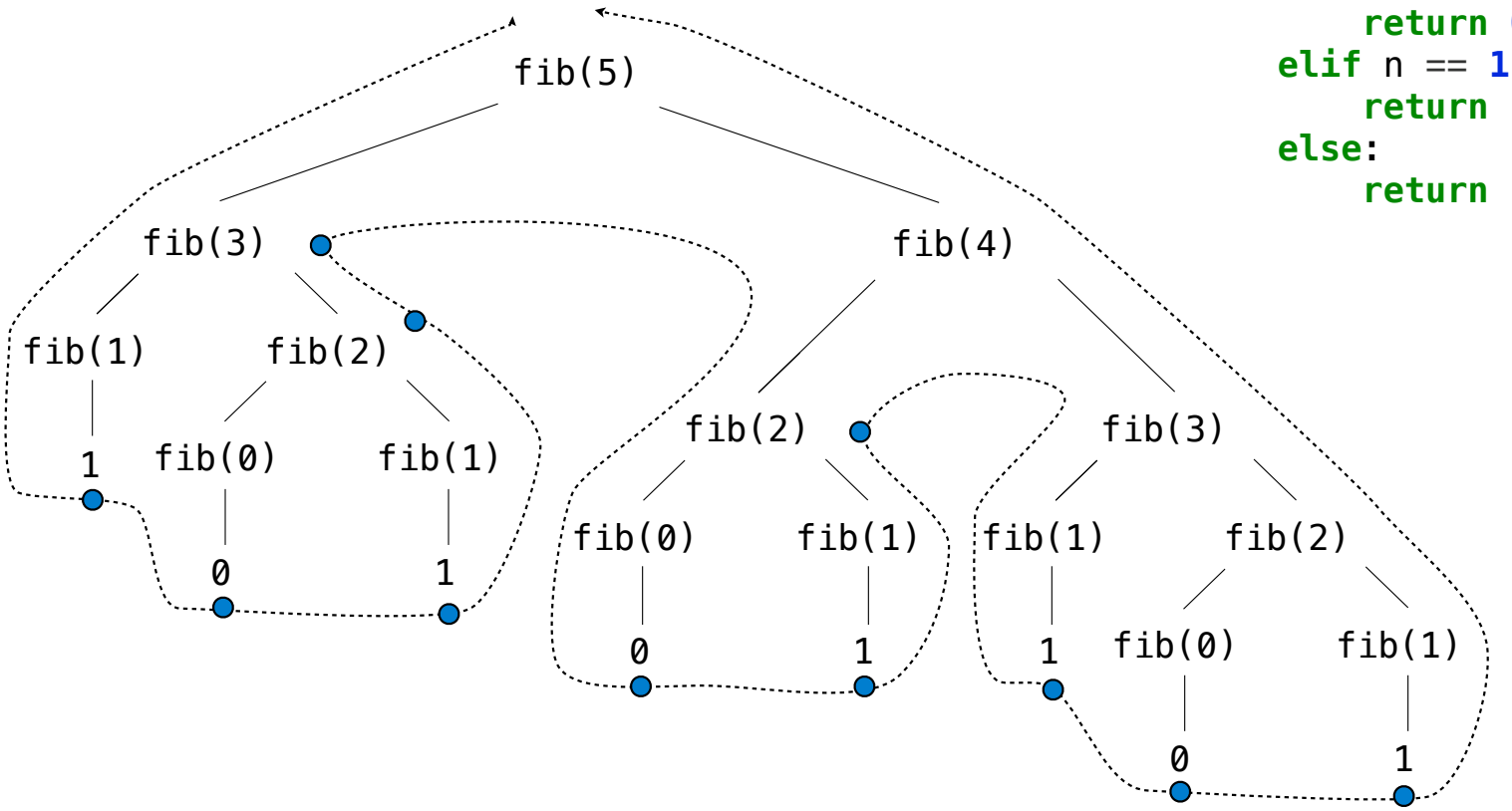

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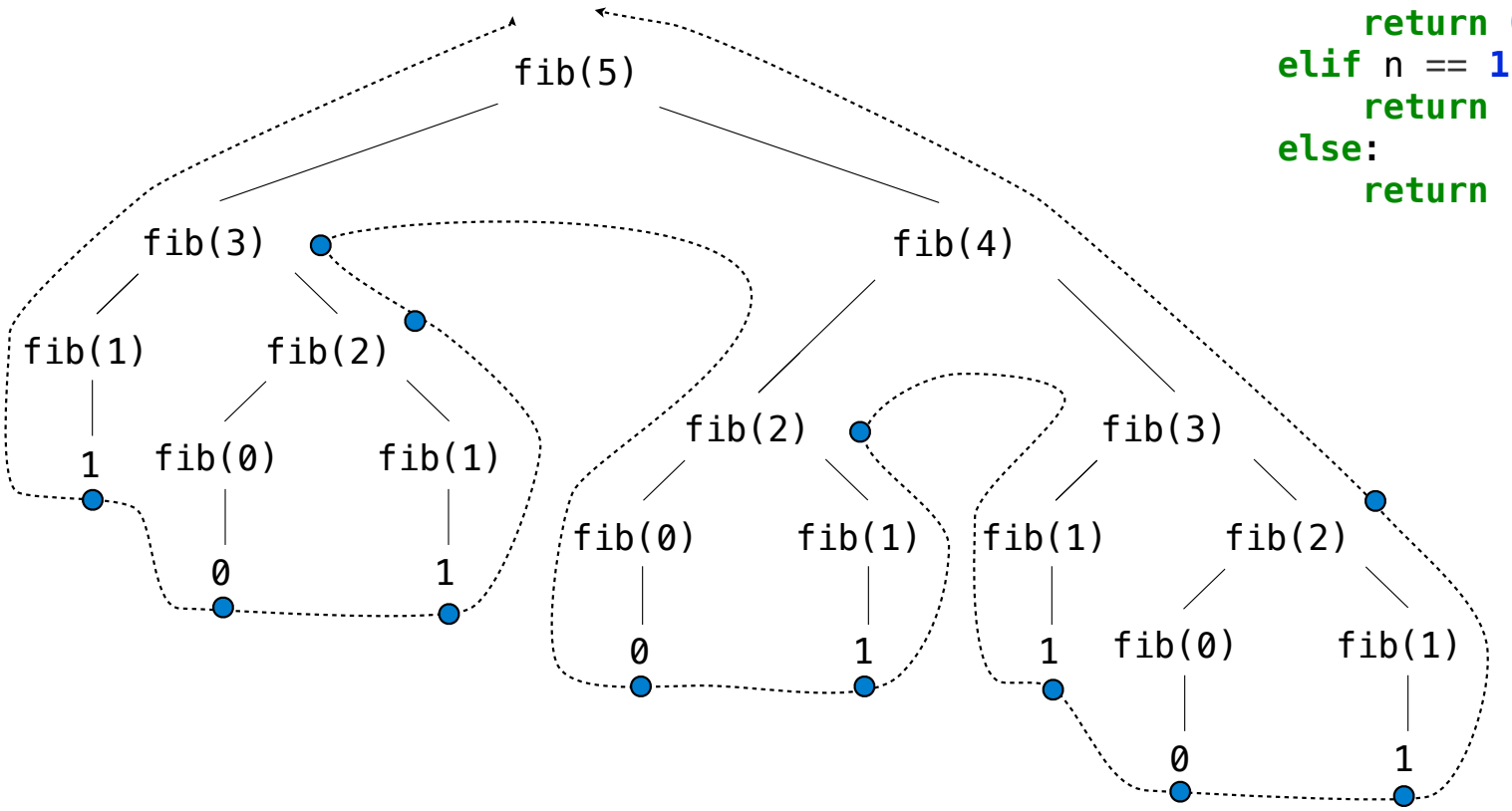

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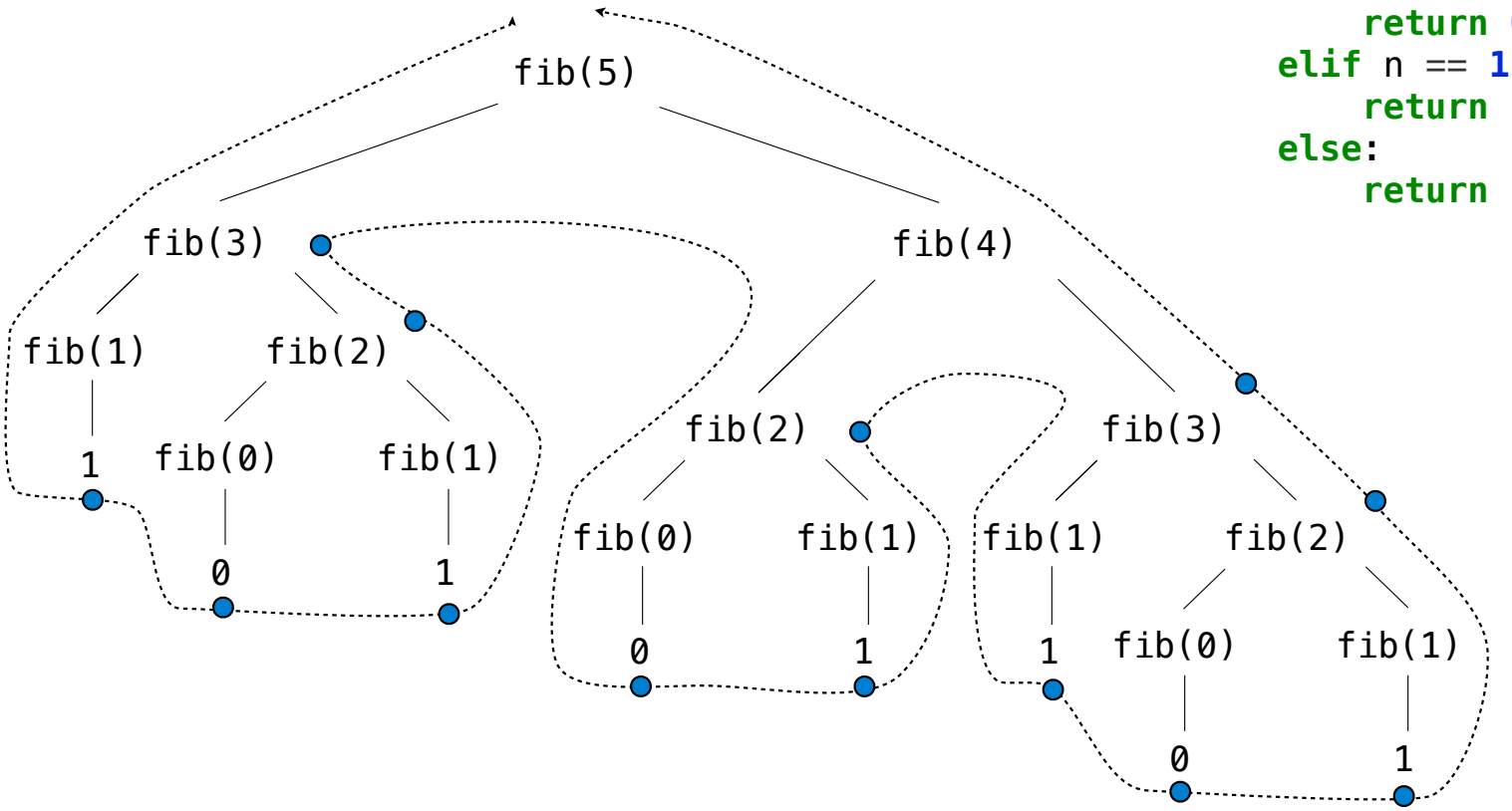

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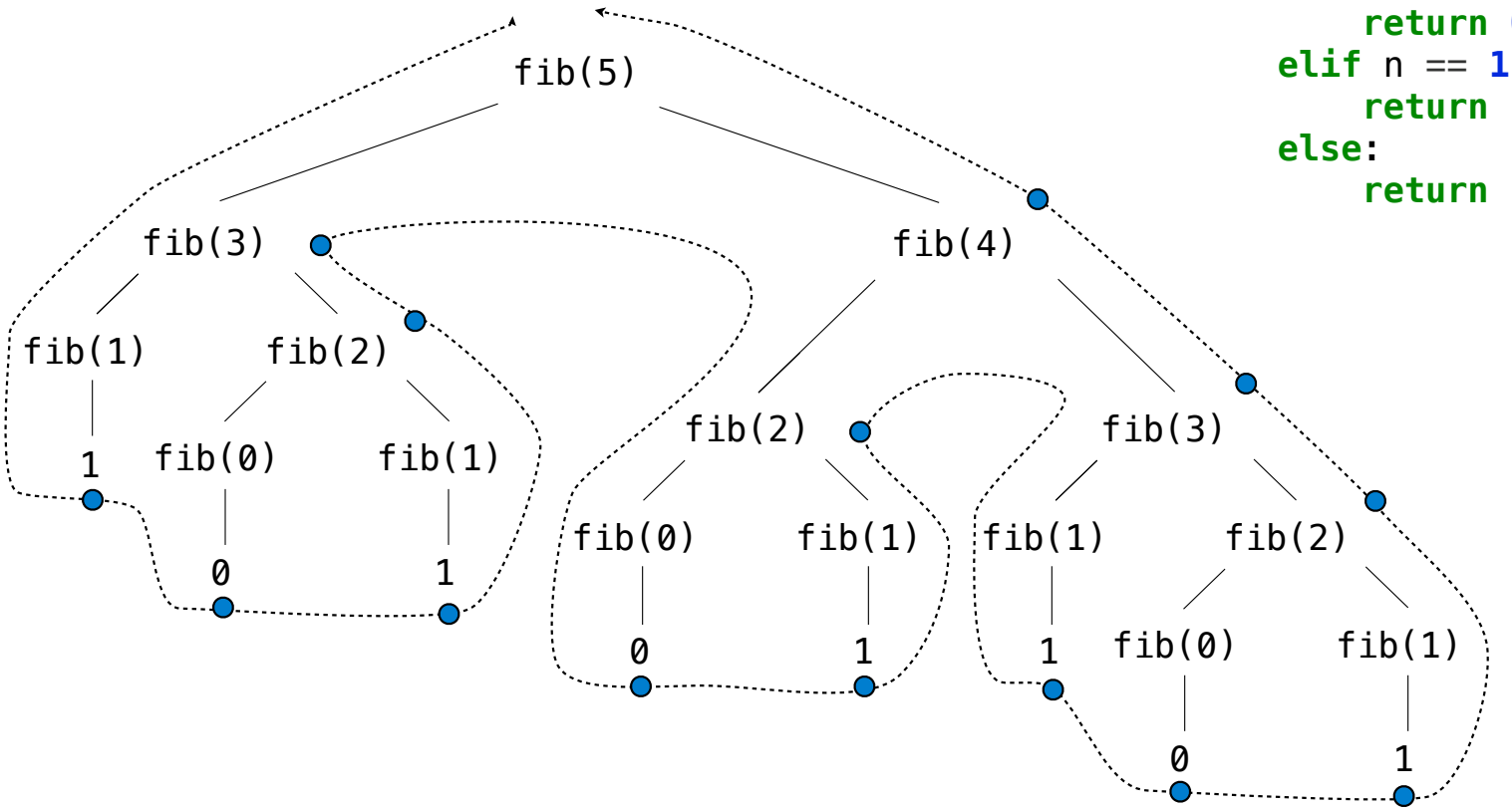

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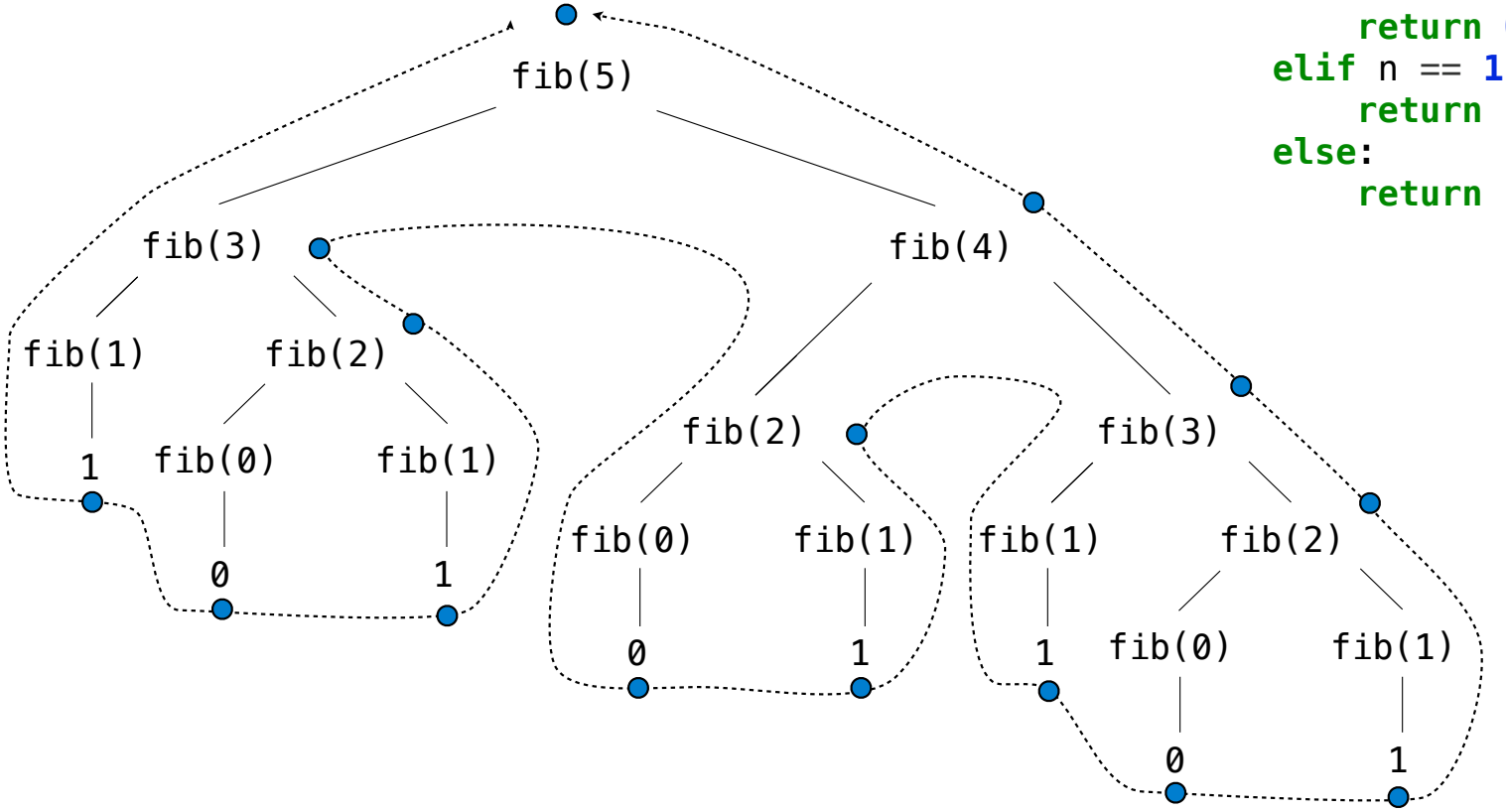


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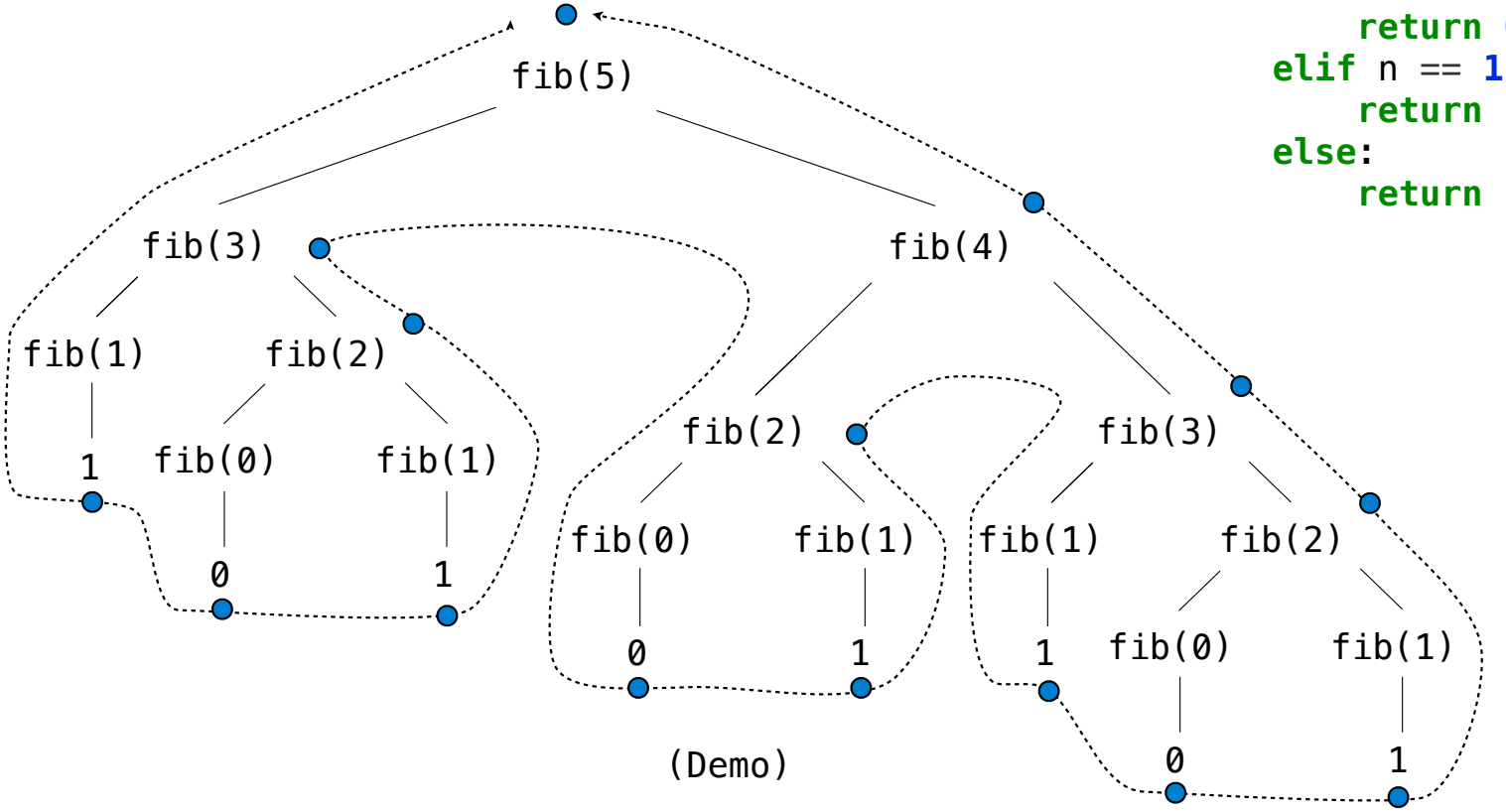


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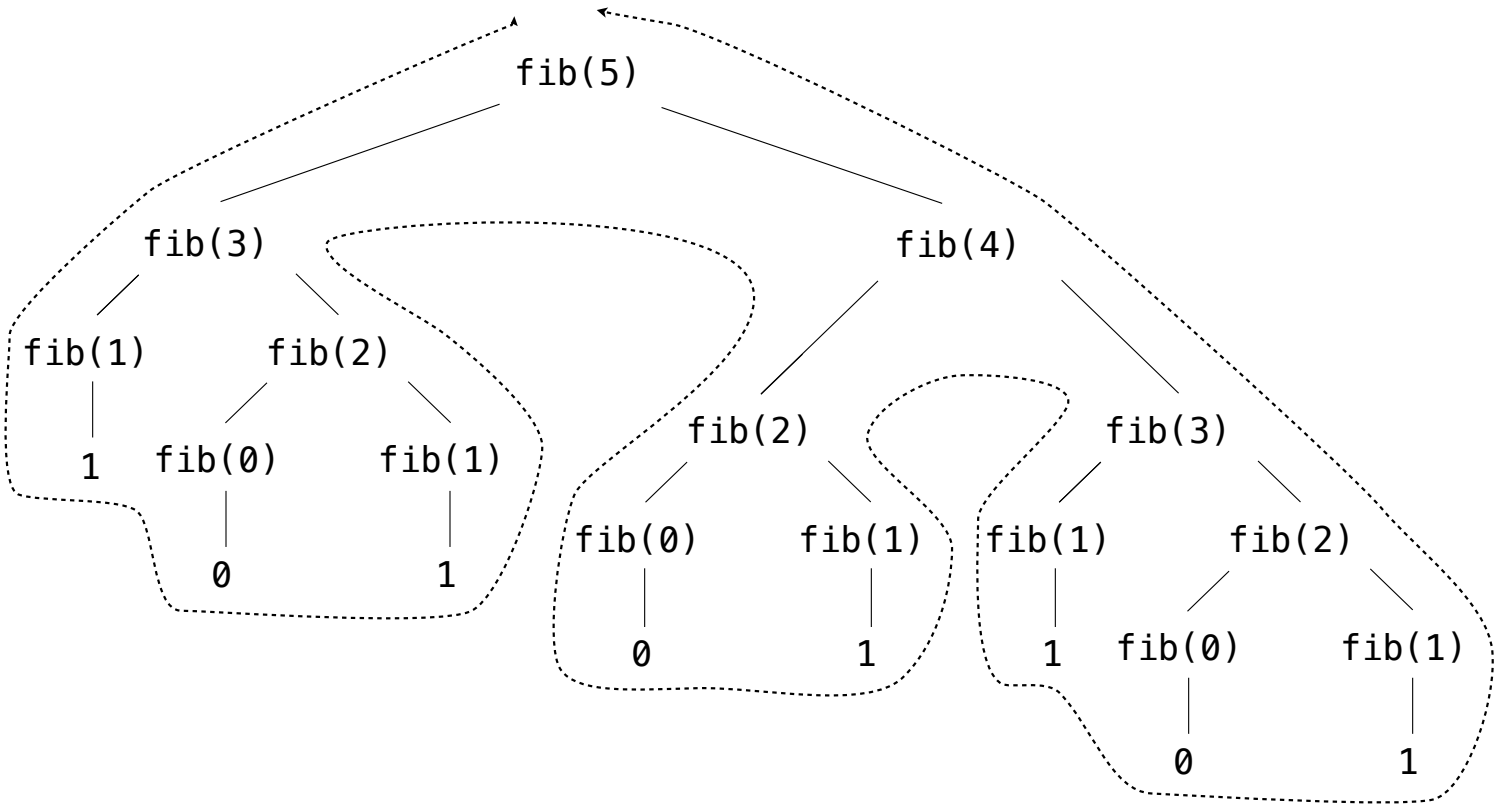
Memoized Tree Recursion



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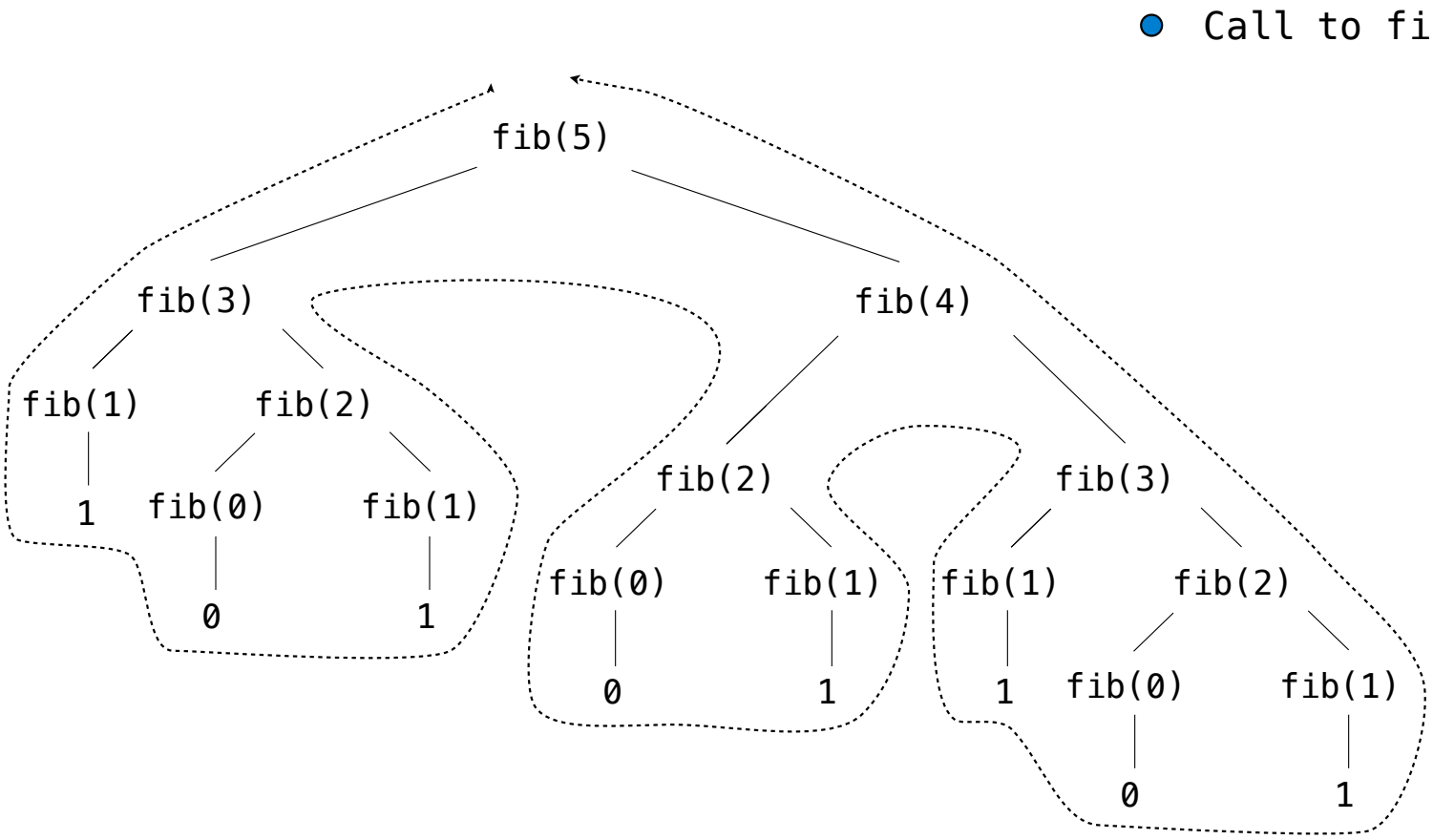

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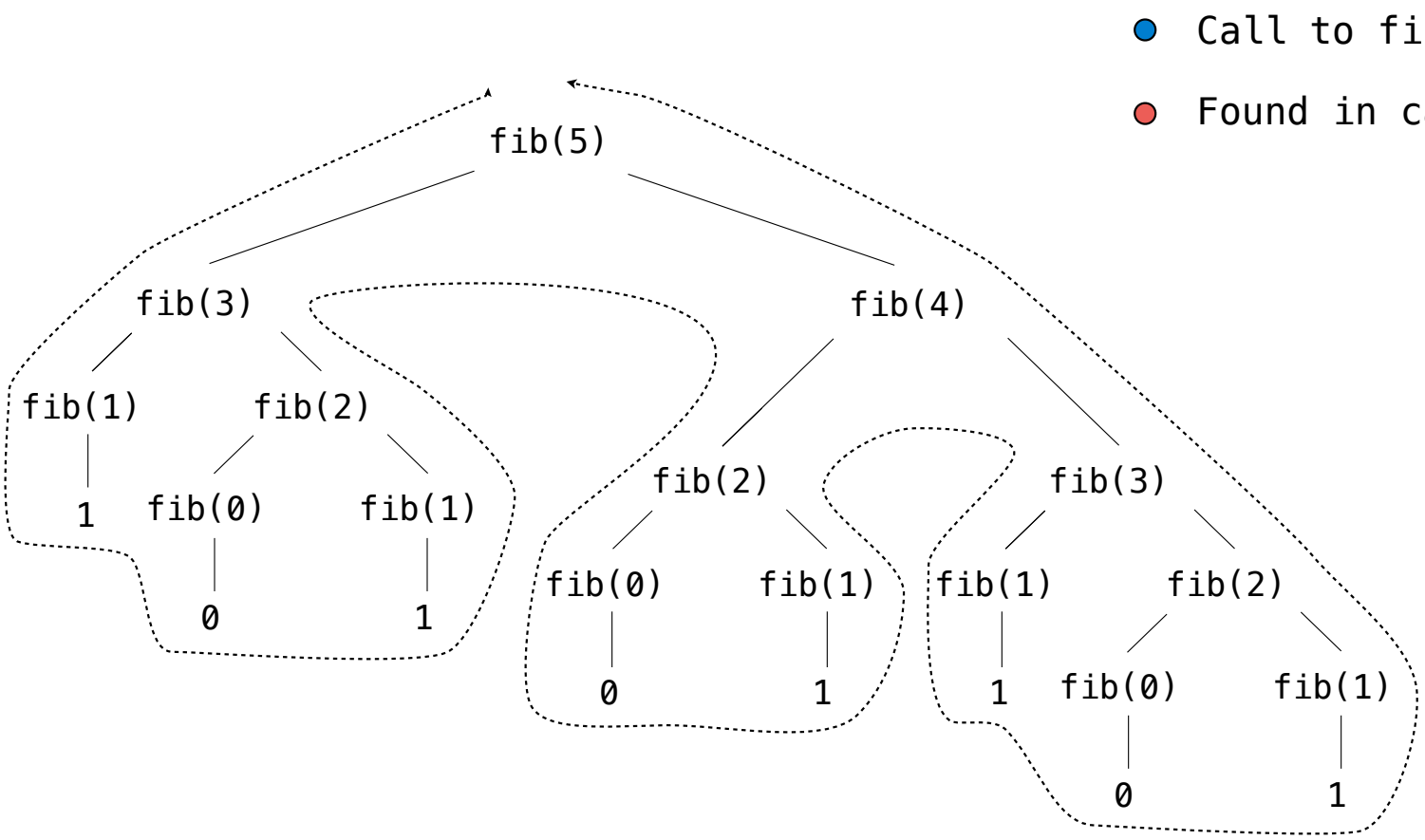

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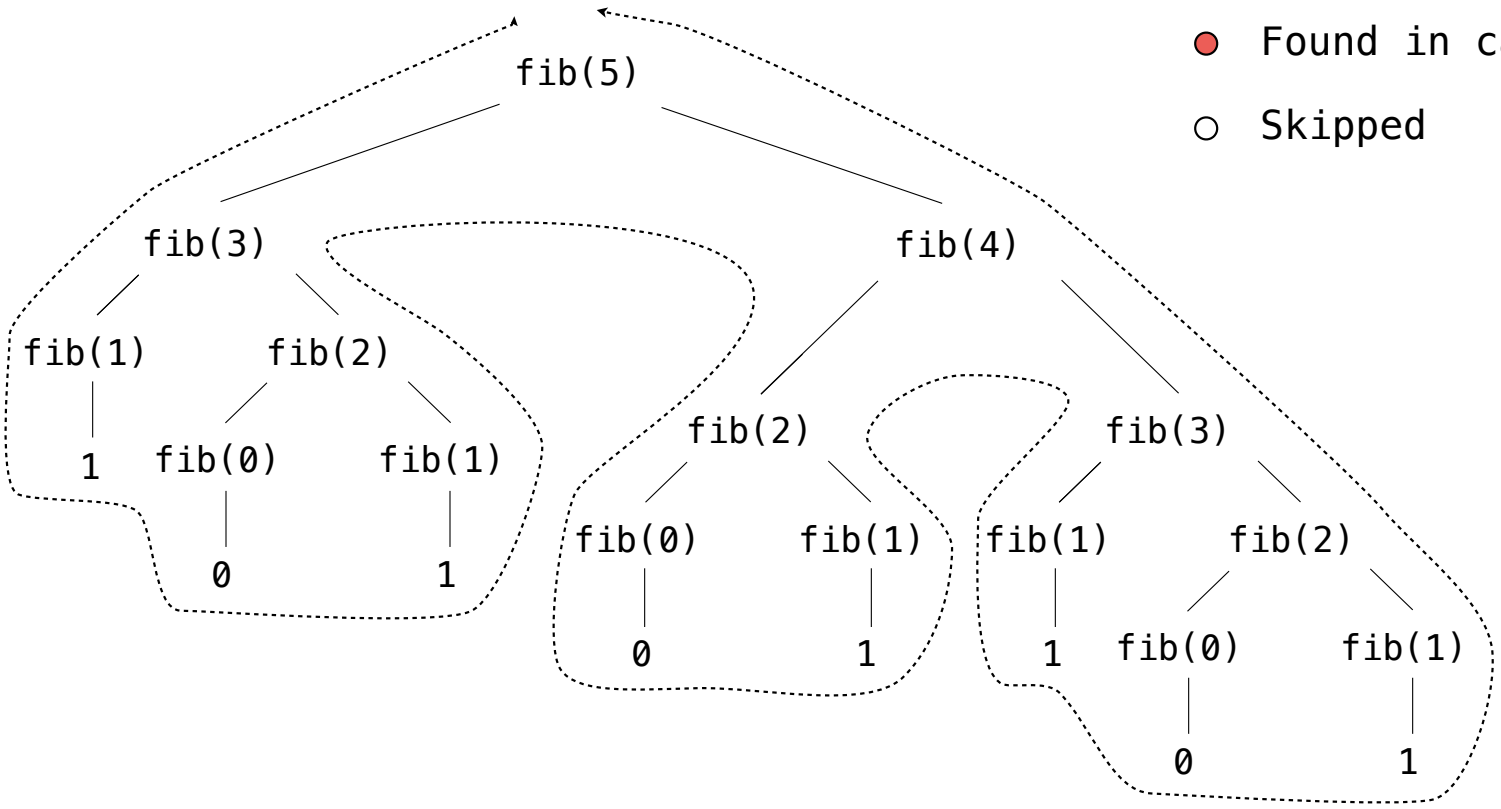

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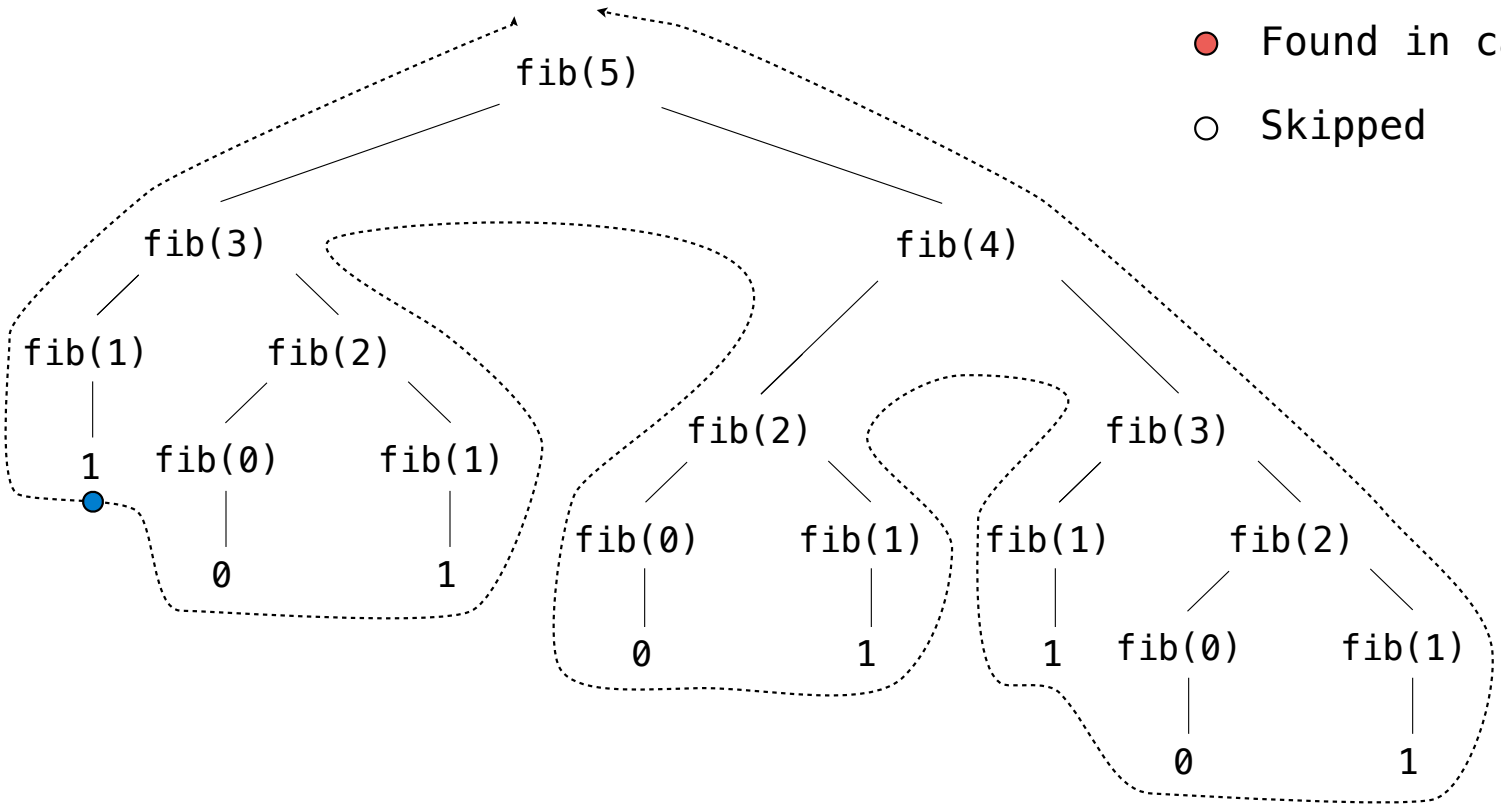

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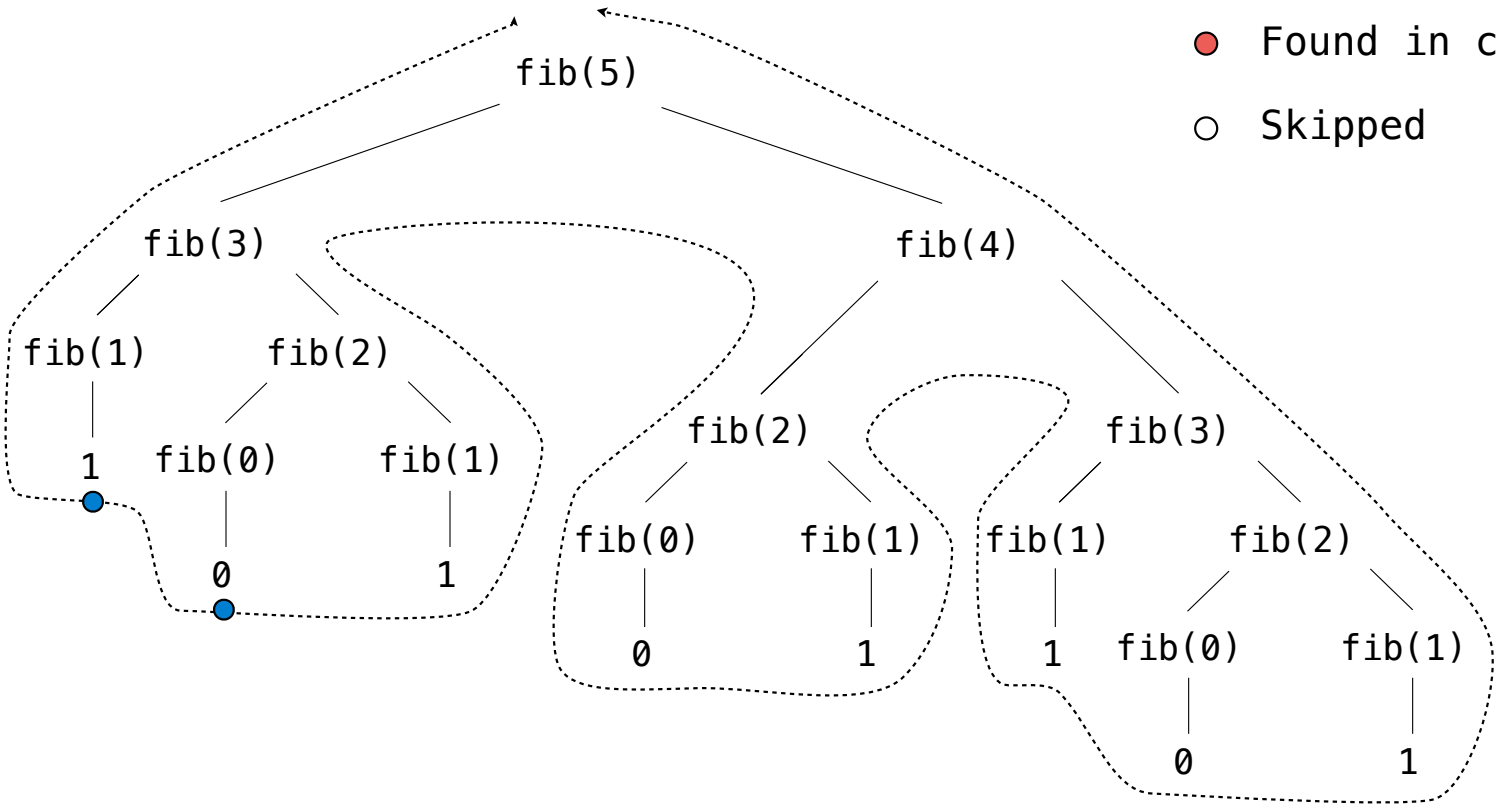

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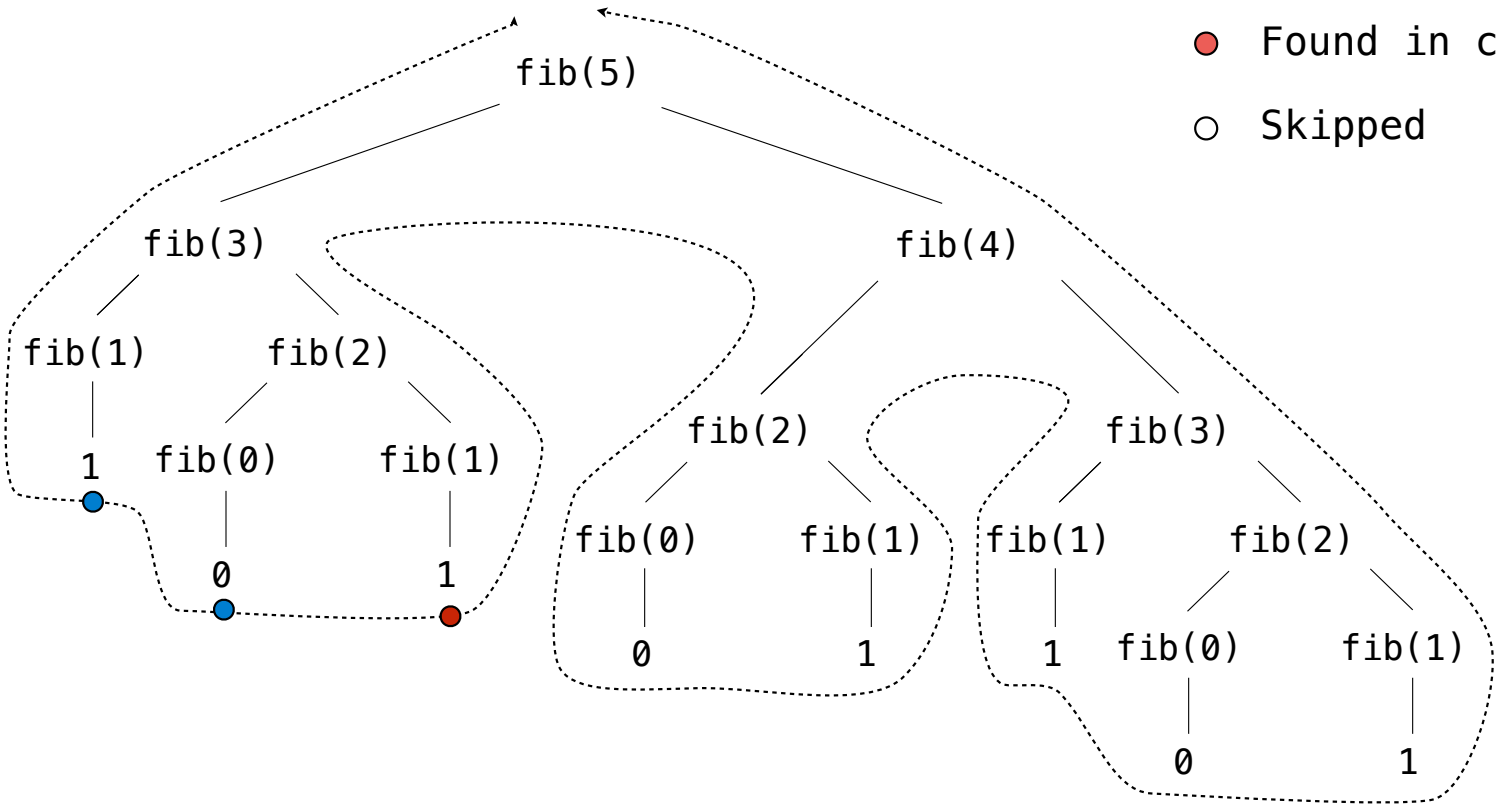

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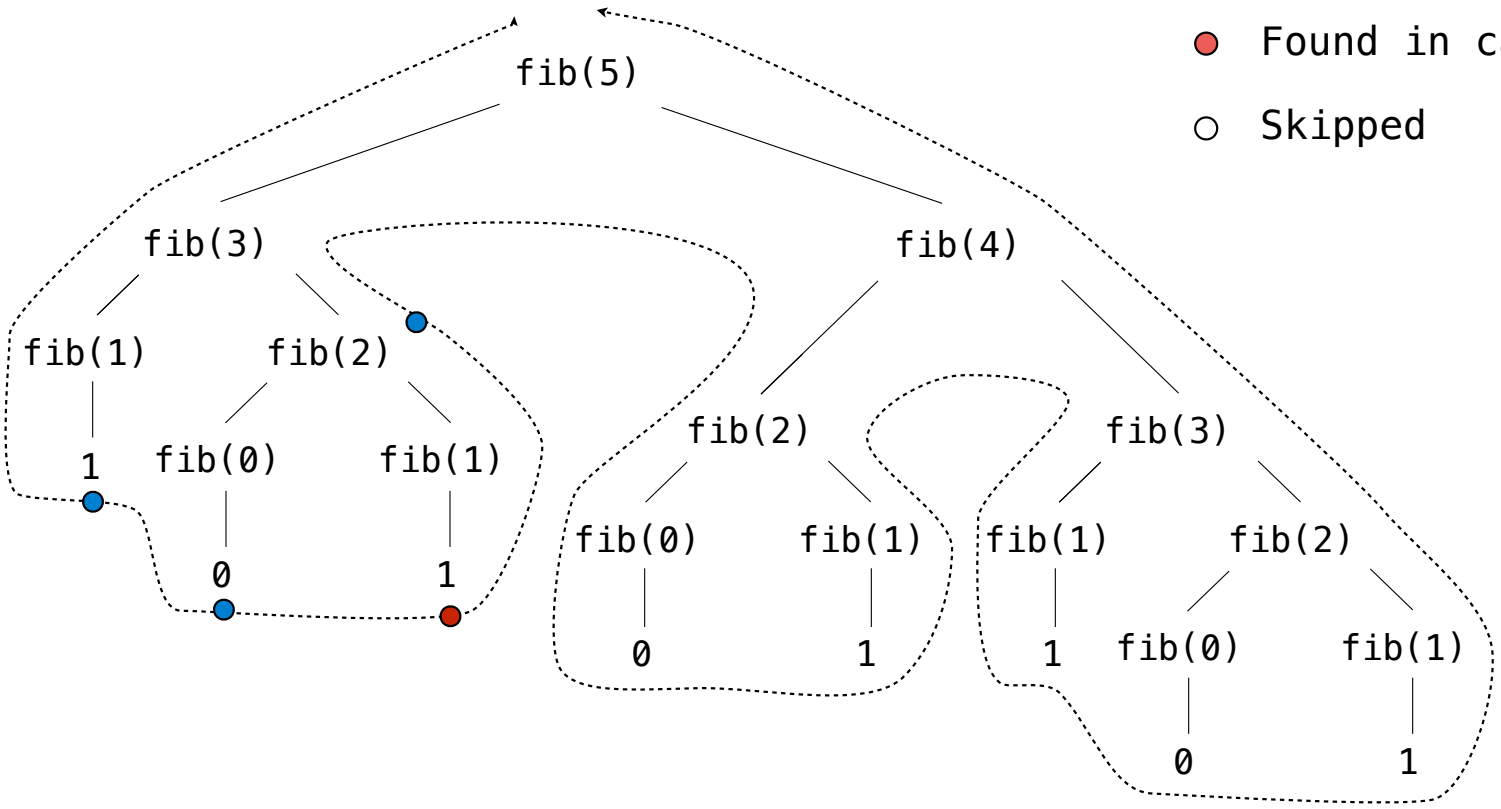

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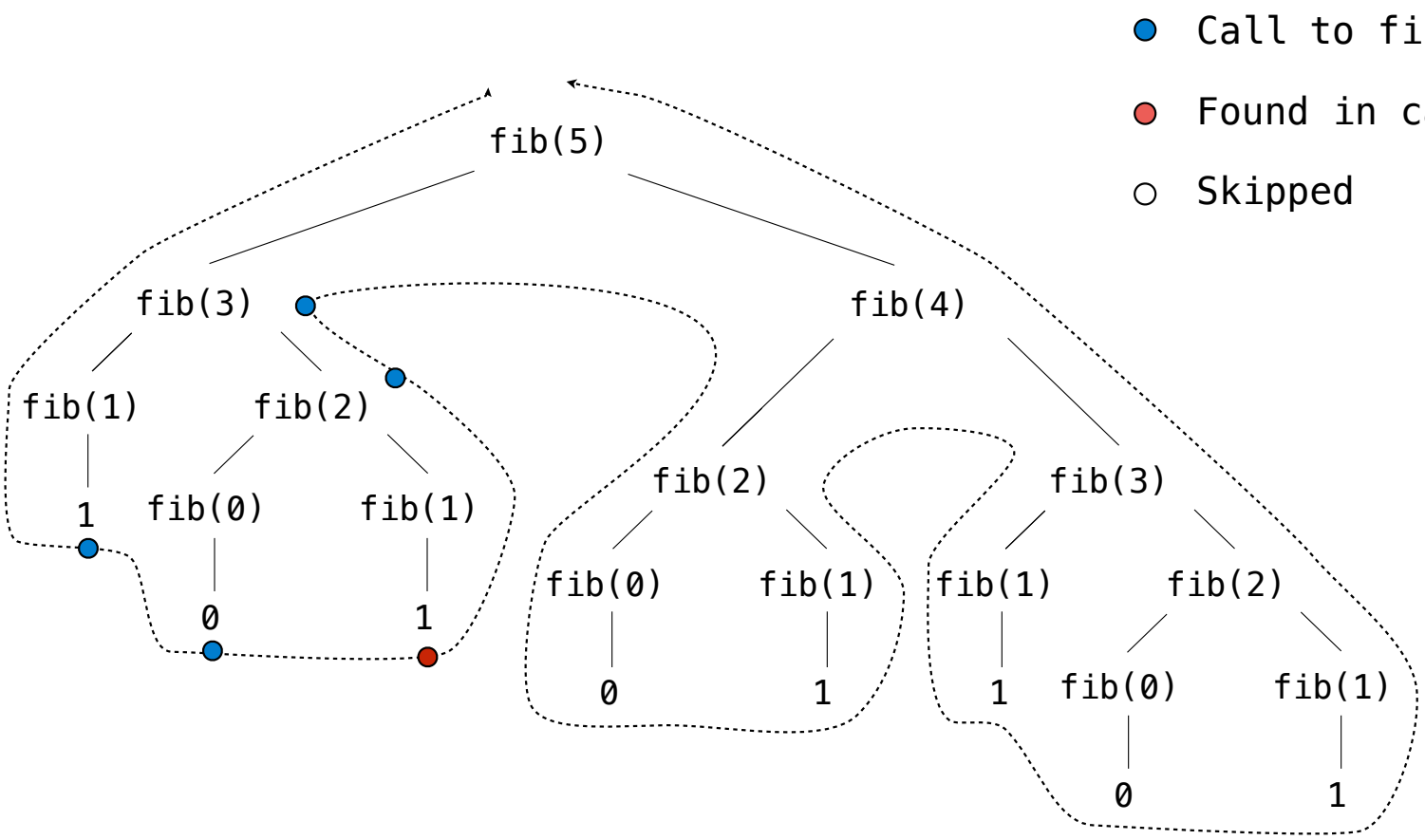

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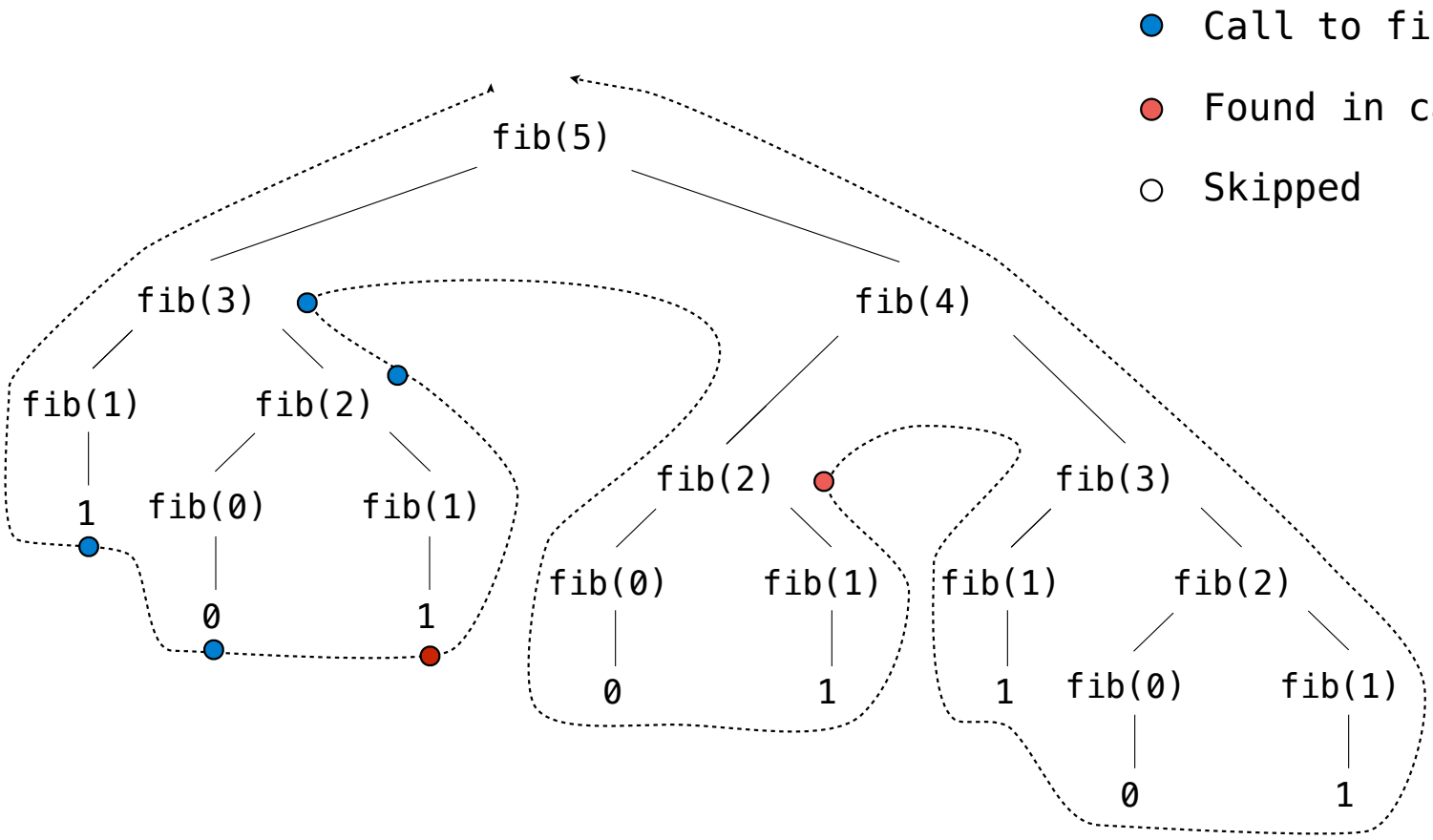

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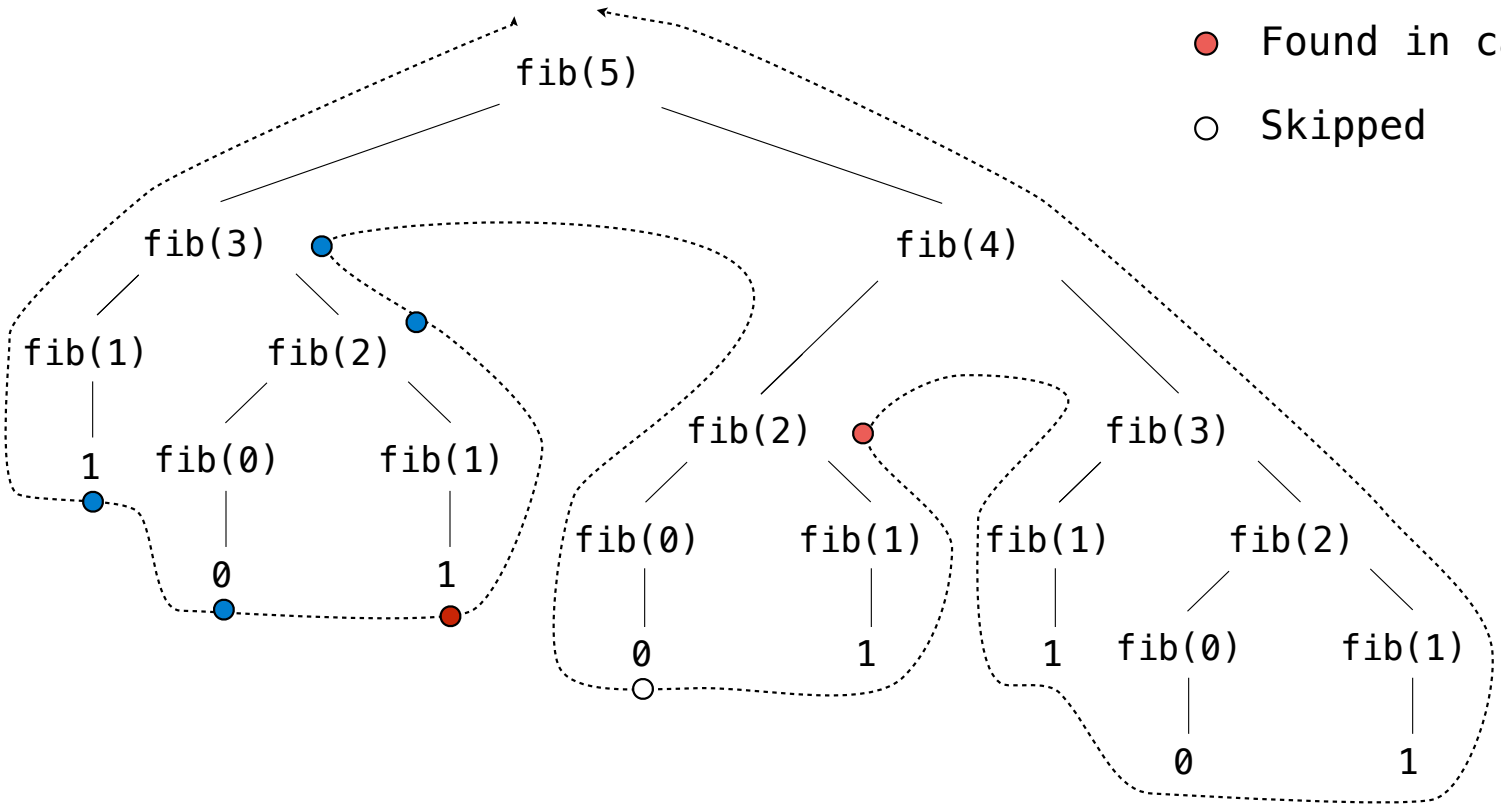

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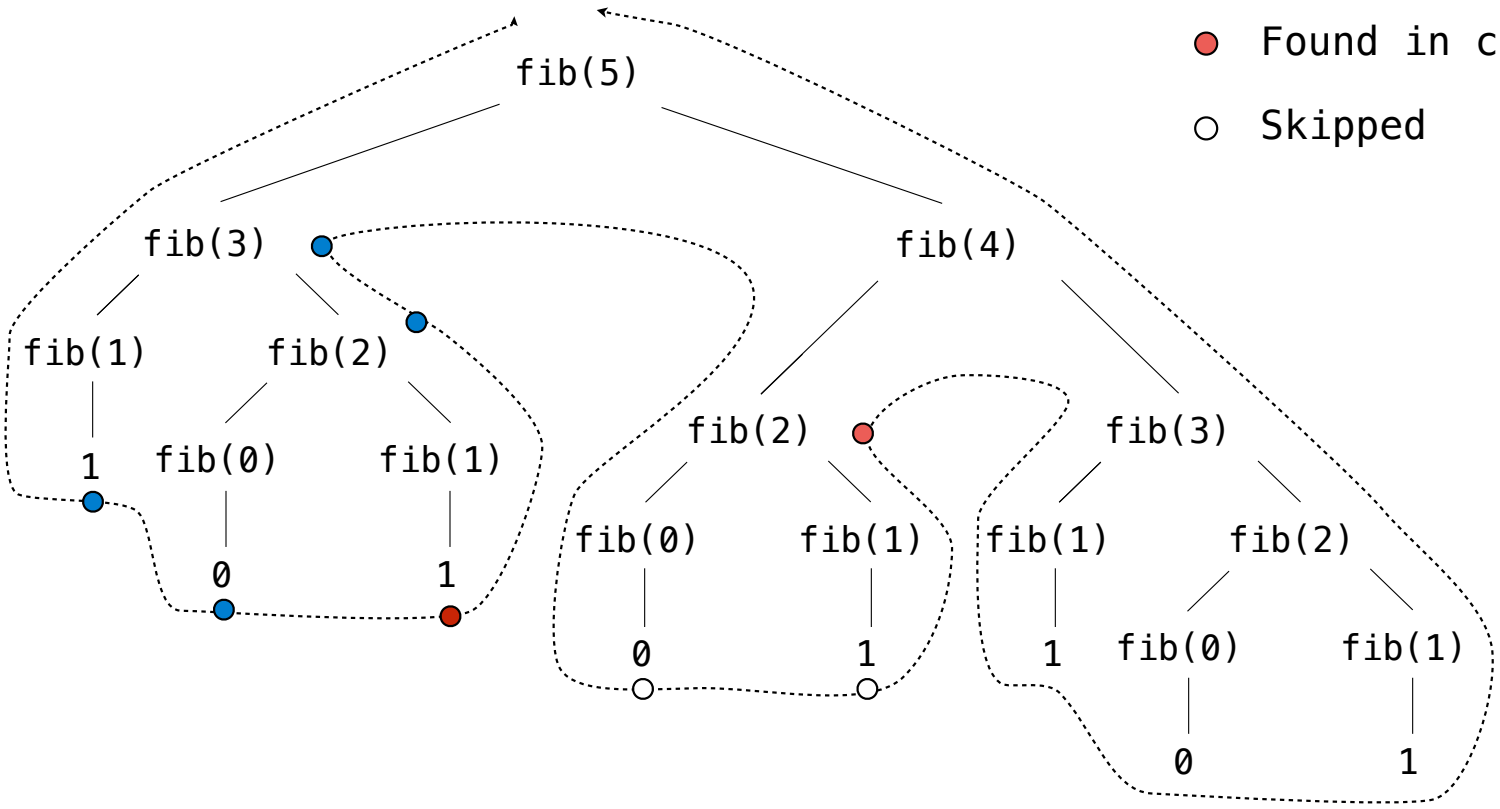

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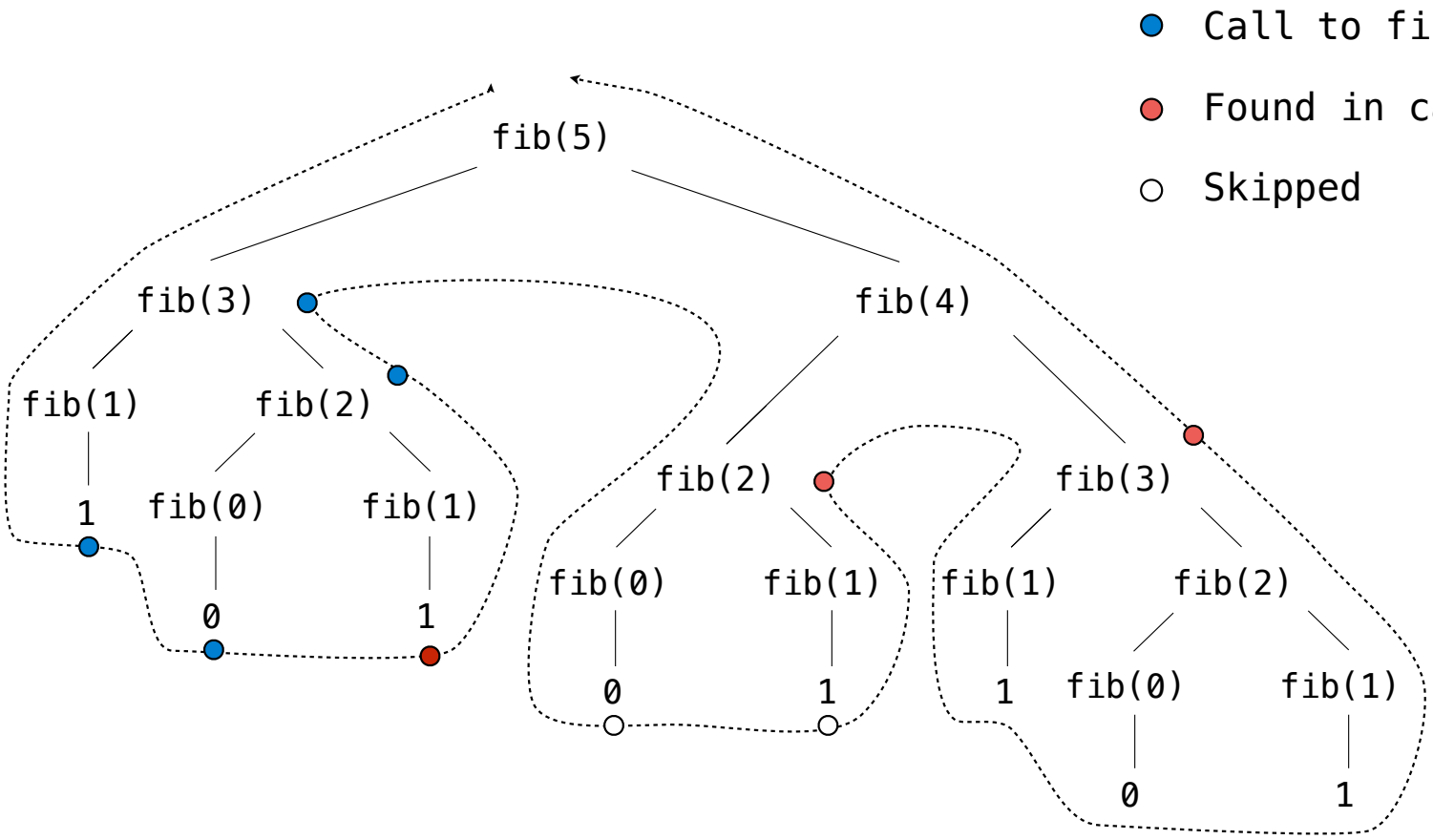

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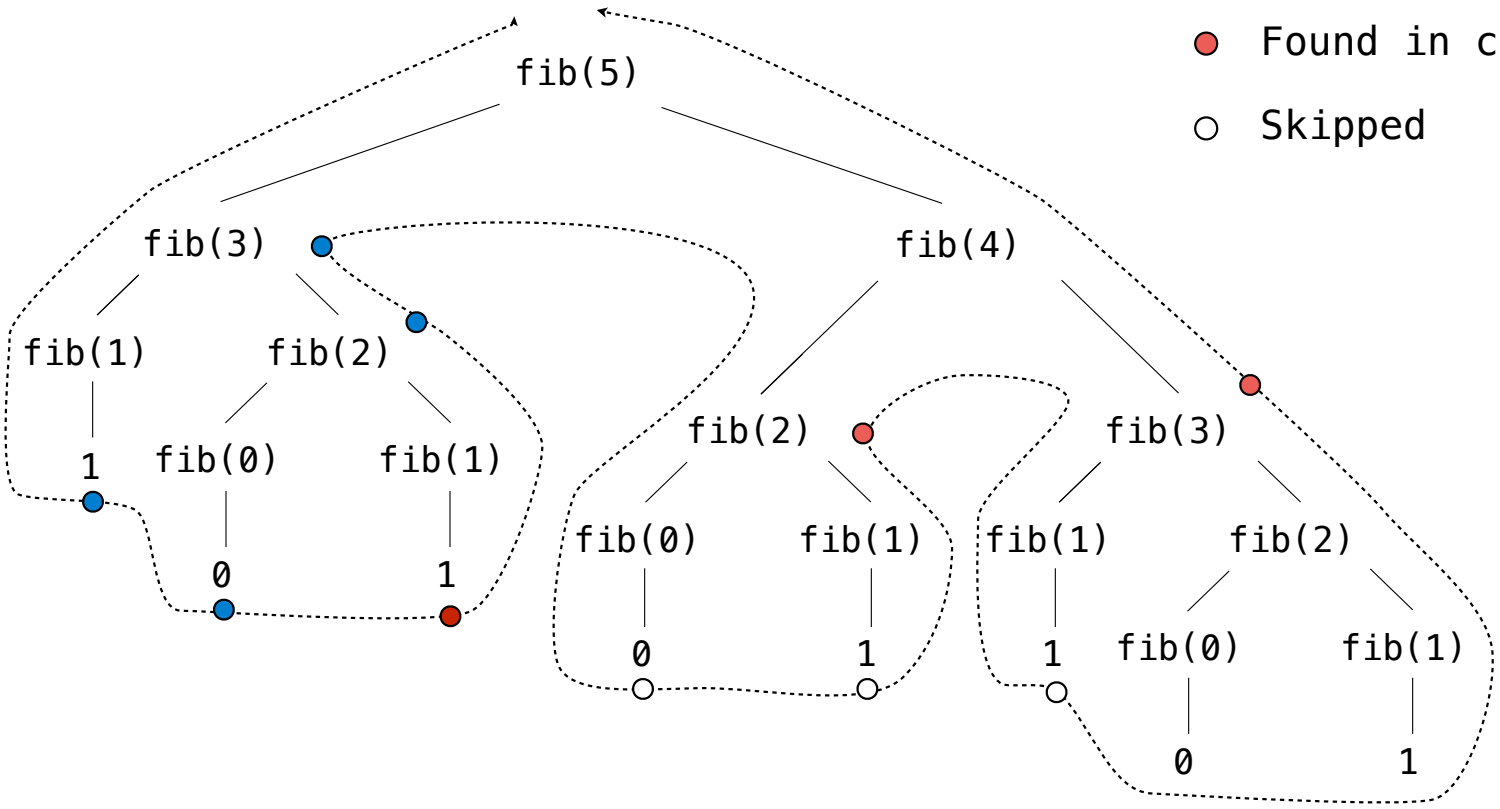

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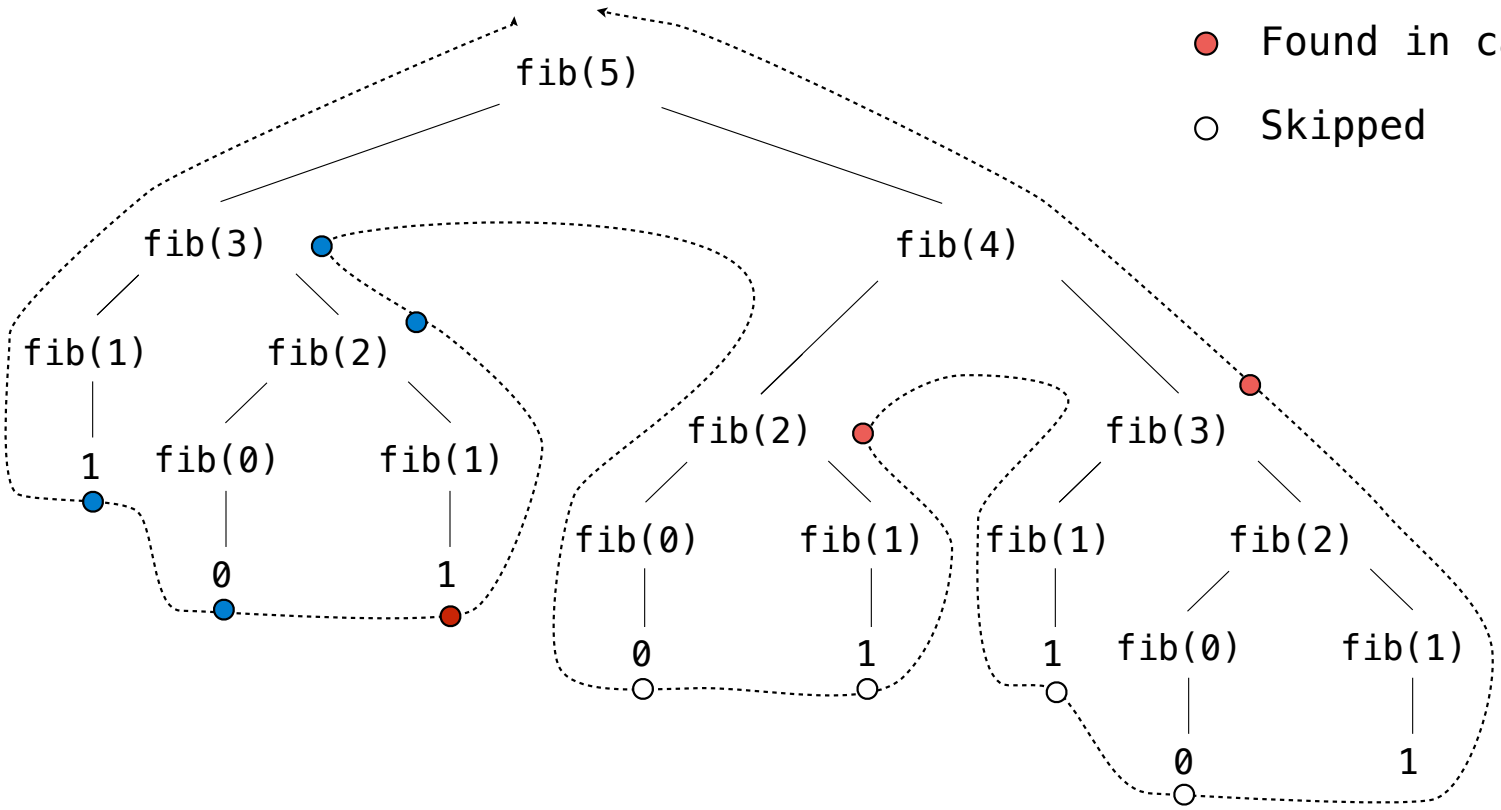

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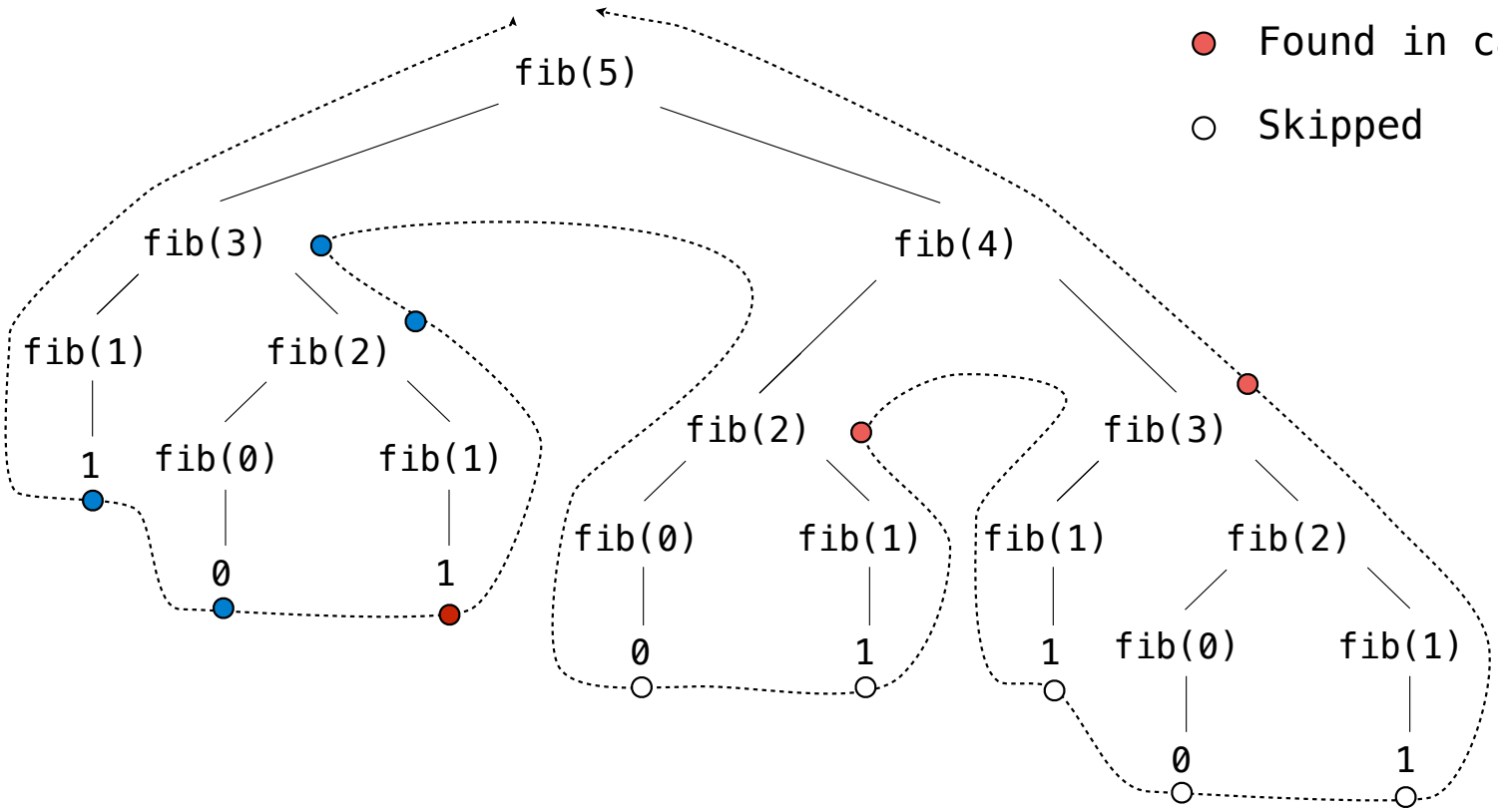

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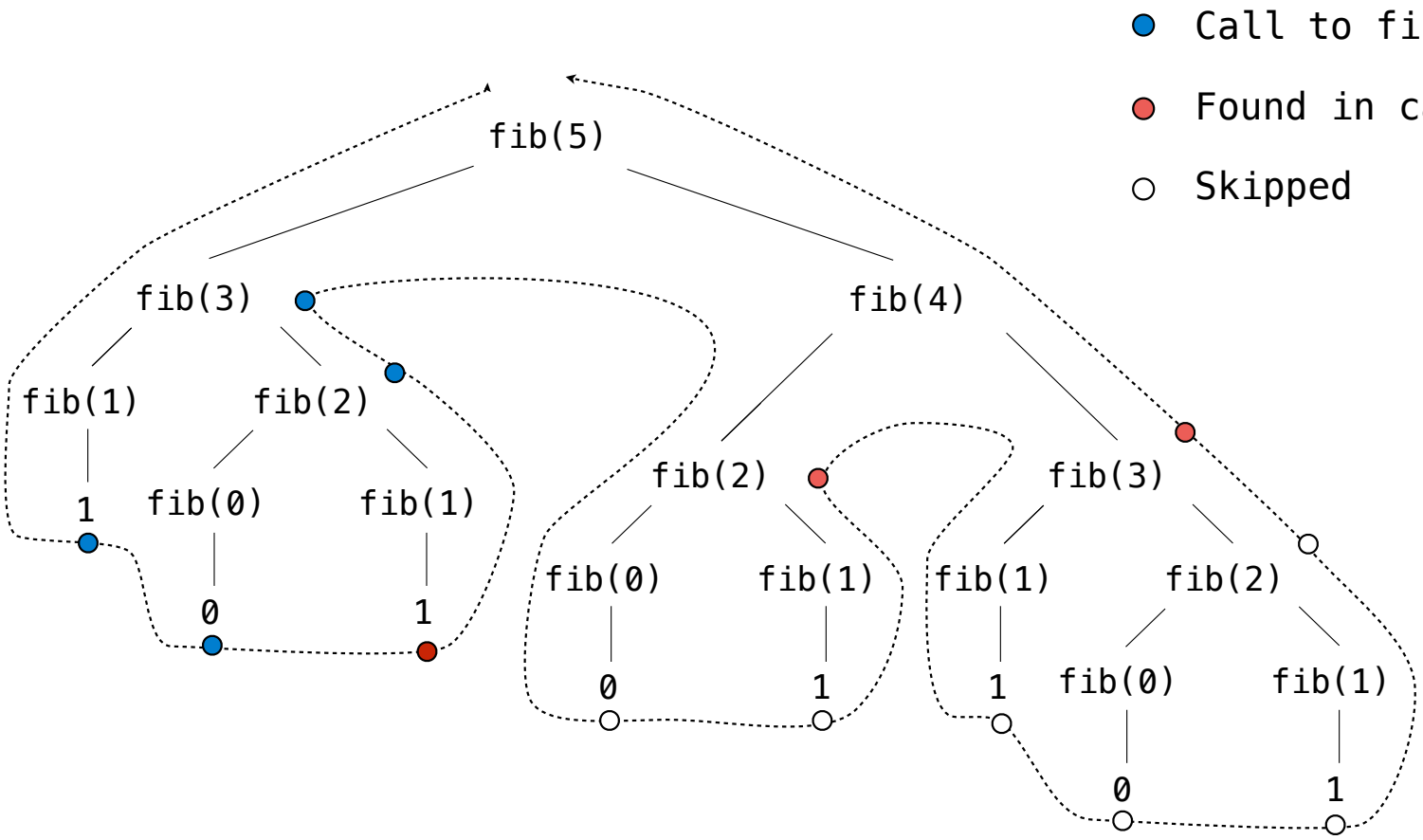

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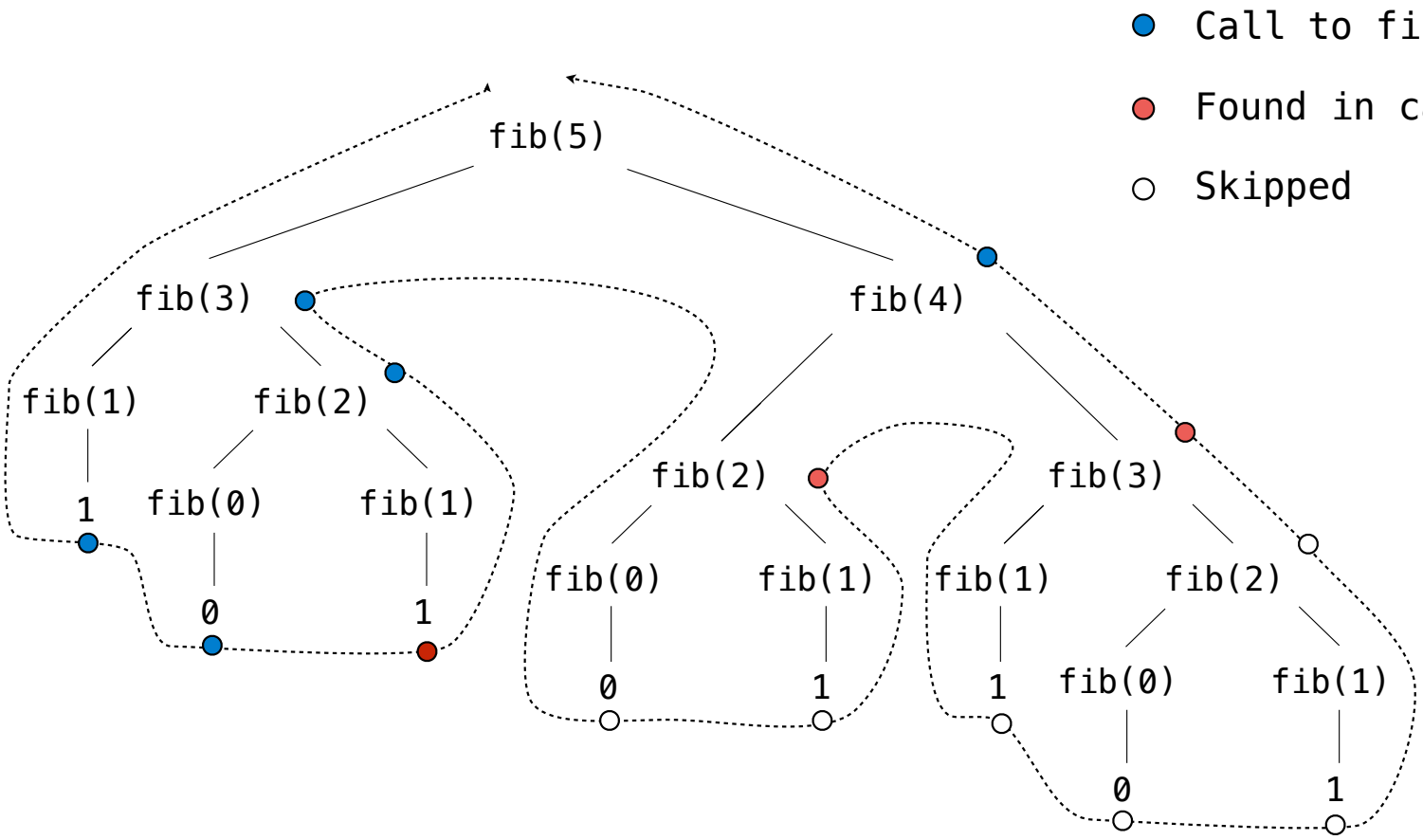

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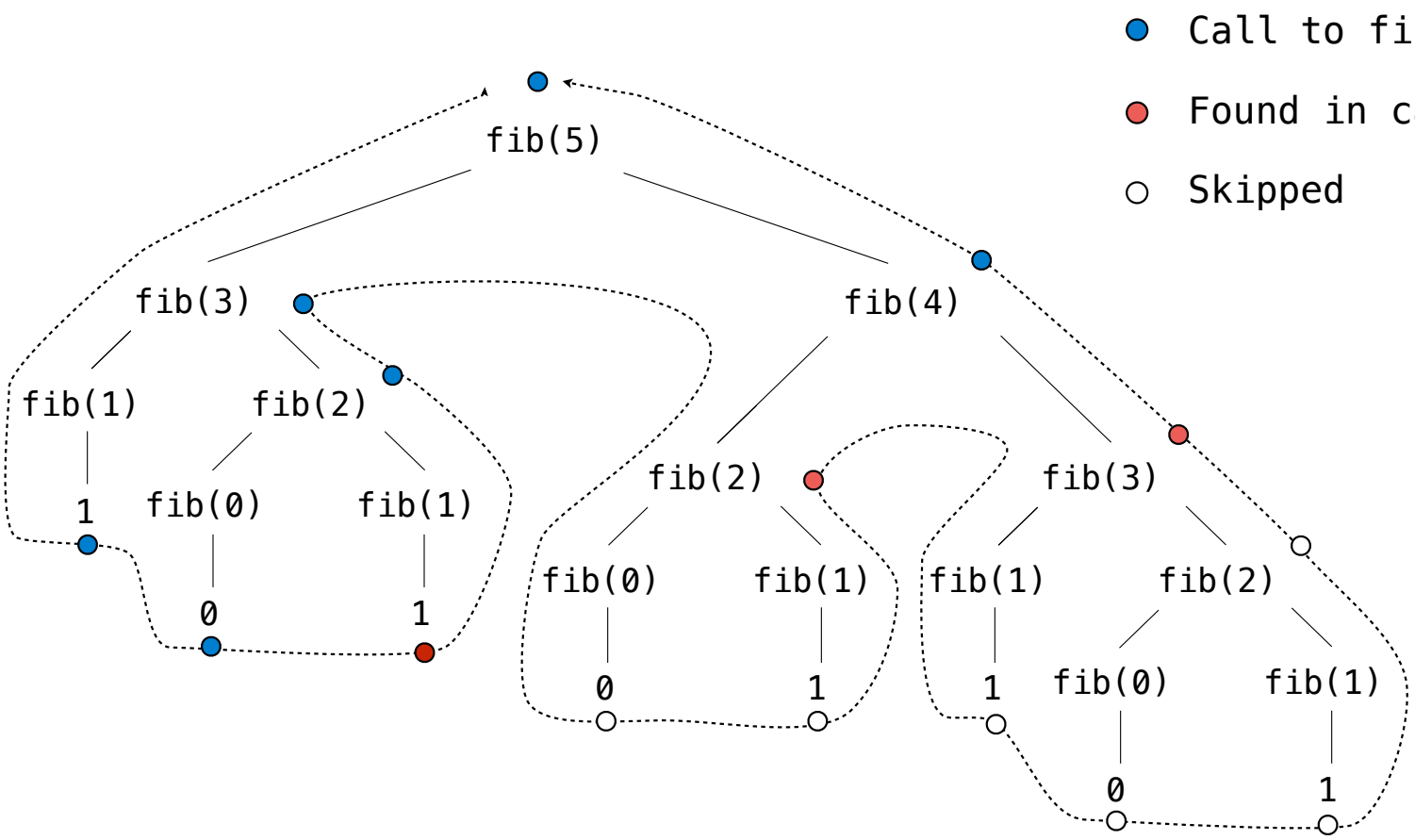

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Twenty-One (Nim)

Twenty-One Rules

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Two players alternate turns, on which they can add 1, 2, or 3 to the current total

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Some states are good; some are bad

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(Demo)

Some states are good; some are bad

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(Demo)

Some states are good; some are bad **21** **20**

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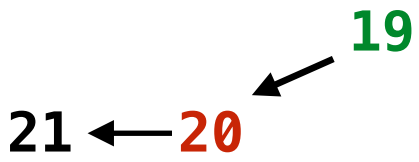

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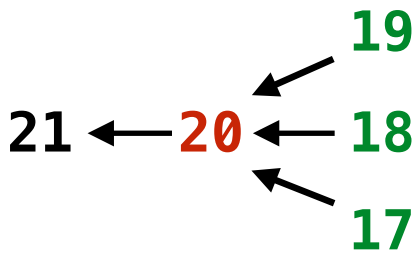

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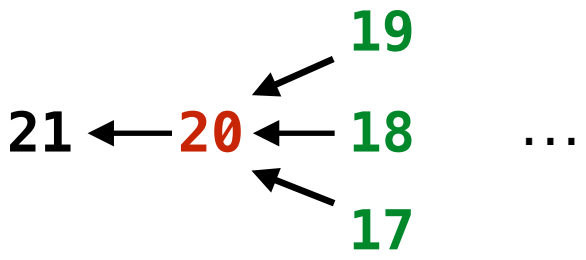

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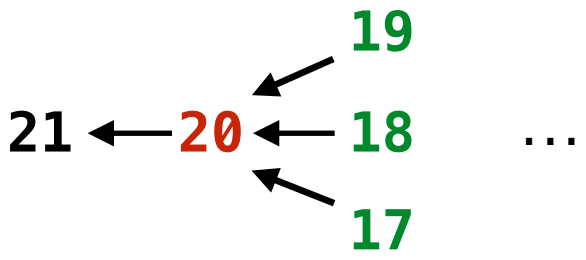

(Demo)

Hog Optimal Strategies

Church Numerals (Homework 2 Challenge Question)