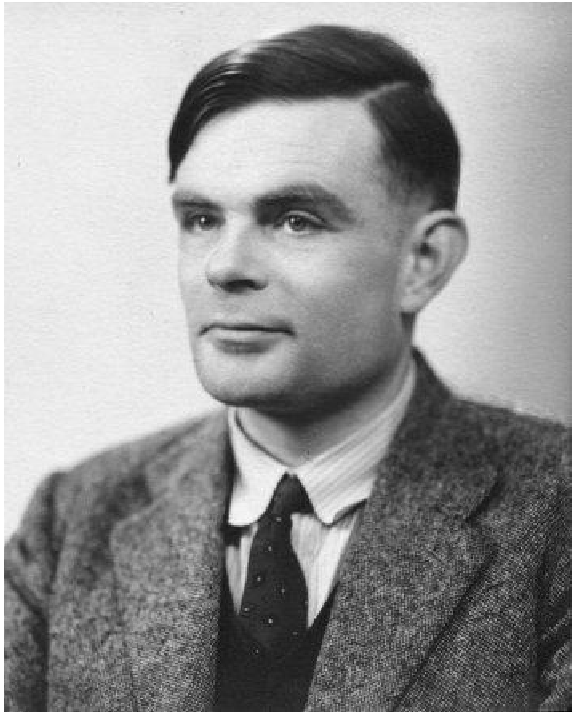
Church-Turing Thesis



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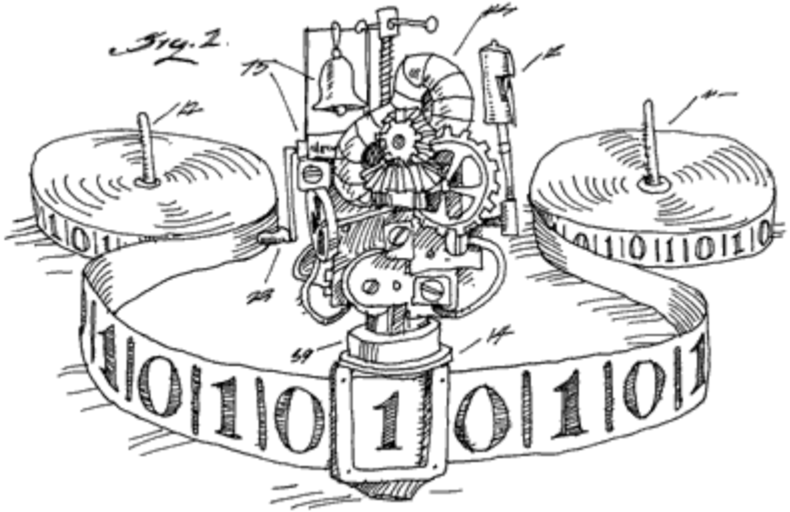

Motivation: Encoding values with functions

```
t = lambda a: lambda b: a
f = lambda a: lambda b: b

def py_pred(p):
    return p(True)(False)

def f_not(p):
    """
    >>> py_pred(f_not(t))
    False
    >>> py_pred(f_not(f))
    True
    """
    return lambda a: lambda b: p(b)(a)
```

Exercise: Implement f_and, f_or

```
f_and = lambda p1: lambda p2:          p1(p2)(f)
f_or  = lambda p1: lambda p2:          p1(t)(p2)
```

A Cleaner Syntax

$$\lambda[\text{argument}].[\text{return value}]$$

$$f(x) \equiv f\,x$$

$$(\lambda x.M)N = M[x := N]$$

Basic Functions

$$I = \lambda t.t$$

$$C_r = \lambda s.r$$

$$C_I = \lambda s.(\lambda t.t)$$

$$K = \lambda r.(\lambda s.r)$$

Exercise:

Confirm that K I is equivalent to C_I

Write down C_{C_I}, also write it in terms of K and I

Evaluate (K K) (K K)

http://www.chenyang.co/lambda

Currying

$$fgh \equiv ((fg)h)$$

$$\lambda x.(\lambda y.xy) \equiv \lambda xy.xy$$

$$\pi_1 = \lambda xy.x$$

$$\pi_2 = \lambda xy.y$$

What does pi_1 and pi_2 remind you of?

True and False

```
true  = lambda a: lambda b: a
false = lambda a: lambda b: b
```

$$T \equiv \lambda ab.a$$
$$F \equiv \lambda ab.b$$

```
f_not = lambda p:  lambda a:
        lambda b:  p(b)(a)
f_and = lambda p1: lambda p2: p1(p2)(false)
f_or  = lambda p1: lambda p2: p1(true)(p2)
```

Exercise:

Translate boolean operators into lambda notation

$$\text{Not} \equiv \lambda pab.pba$$
$$\text{And} \equiv \lambda pq.pqF$$
$$\text{Or} \equiv \lambda pq.pTq$$

Write an if-function using lambda notation

Zero, One, Two, ...

$$nf \to f^n = f \circ f \circ \cdots \circ f$$
$$= \lambda x.f(f(\cdots f(x)\cdots))$$
$$n = \lambda fx.f(f(\cdots f(x)\cdots))$$

$$0 \equiv \lambda fx.x$$
$$1 \equiv \lambda fx.fx$$
$$2 \equiv \lambda fx.f(fx)$$

$$Sn = \lambda fx.f^{n+1}x$$
$$= \lambda fx.f(f^n x)$$
$$= \lambda fx.f(nfx)$$
$$S = \lambda nfx.f(nfx)$$

```
Exercise:

Implement add, mul, power

Suppose we have an operator pred that
returns the predecessor of a number,
implement subtraction using pred

Implement pred (Challenging)
```

Recursion!

```
fac = lambda n: 1 if n == 0 else n * fac(n-1)

F = lambda fac: lambda n: 1 if n == 0 else n * fac(n-1)

F(fac) = fac  <- fac is the fixed point of F
```

$$F(YF) = YF$$

Fixed Points

$$F(YF) = YF$$

$$\omega = (\lambda x.xx)(\lambda x.xx)$$

$$\begin{aligned}\omega' &= (\lambda x.F(xx))(\lambda x.F(xx)) \\ &= F((\lambda x.F(xx))(\lambda x.F(xx))) = F(\omega')\end{aligned}$$

$$\boxed{Y = \lambda F.(\lambda x.F(xx))(\lambda x.F(xx))}$$

Puzzle

What is X? (There are 26 Ls)

Hint: A fixed point combinator has the form YF=F(YF)

$$X = (LLLLLLLLLLLLLLLLLLLLLLLLLL)$$

$$L = \lambda abcdefghijklmnopqstuvwxyzr.r(thisisafixedpointcombinator)$$

$$X = \lambda r.r(Xr)$$

$$XF = F(XF)$$

Conclusion + Acknowledgements

Everything + more:

http://xuanji.appspot.com/isicp/lambda.html

Lambda interpreter:

https://github.com/tarao/LambdaJS