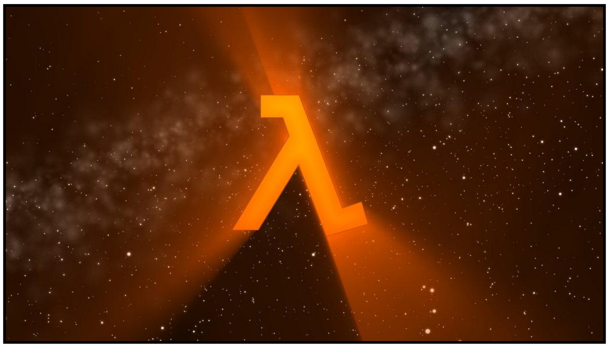
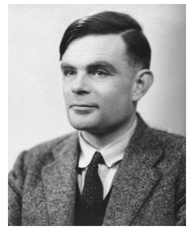
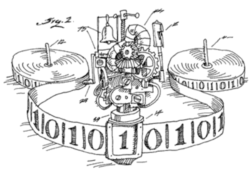


Church-Turing Thesis



Church-Turing Thesis



Motivation: Encoding values with functions

```python
t = lambda a: lambda b: a
f = lambda a: lambda b: b

def py_pred(p):
    return p(True)(False)

def f_not(p):
    """
    >>> py_pred(f_not(t))
    False
    >>> py_pred(f_not(f))
    True
    """
    return lambda a: lambda b: p(b)(a)
```

```
Exercise: Implement f_and, f_or
f_and = lambda p1: lambda
p2:                              p1(p2)(f)
f_or  = lambda p1: lambda       p1(t)(p2)
p2:
```

A Cleaner Syntax

$$\lambda[\text{argument}].[\text{return value}]$$

$$f(x) \equiv fx$$

$$(\lambda x.M)N = M[x := N]$$

Basic Functions

$$I = \lambda t.t$$

$$C_r = \lambda s.r$$

$$C_I = \lambda s.(\lambda t.t)$$

$$K = \lambda r.(\lambda s.r)$$

Exercise:

Confirm that K I is equivalent to C_I

Write down C_{C_I}, also write it in terms of K and I

Evaluate (K K) (K K)

http://www.chenyang.co/lambda

Currying

$$fgh \equiv ((fg)h)$$
$$\lambda x.(\lambda y.xy) \equiv \lambda xy.xy$$
$$\pi_1 = \lambda xy.x$$
$$\pi_2 = \lambda xy.y$$

What does pi_1 and pi_2 remind you of?

True and False

```
true  = lambda a: lambda b: a
false = lambda a: lambda b: b
```

$$T \equiv \lambda ab.a$$
$$F \equiv \lambda ab.b$$

```
f_not = lambda p:  lambda a:
        lambda b:  p(b)(a)
f_and = lambda p1: lambda p2: p1(p2)(false)
f_or  = lambda p1: lambda p2: p1(true)(p2)
```

Exercise:

Translate boolean operators into lambda notation

$$\text{Not} \equiv \lambda pab.pba$$
$$\text{And} \equiv \lambda pq.pqF$$
$$\text{Or} \equiv \lambda pq.pTq$$

Write an if-function using lambda notation

Zero, One, Two, ...

$$\begin{aligned} nf \rightarrow f^n &= f \circ f \circ \cdots \circ f \\ &= \lambda x.f(f(\cdots f(x)\cdots)) \\ n &= \lambda fx.f(f(\cdots f(x)\cdots)) \end{aligned}$$

$$0 \equiv \lambda fx.x$$
$$1 \equiv \lambda fx.fx$$
$$2 \equiv \lambda fx.f(fx)$$

$$\begin{aligned} Sn &= \lambda fx.f^{n+1}x \\ &= \lambda fx.f(f^n x) \\ &= \lambda fx.f(nfx) \\ S &= \lambda nfx.f(nfx) \end{aligned}$$

Exercise:

Implement add, mul, power

Suppose we have an operator pred that returns the predecessor of a number, implement subtraction using pred

Implement pred (Challenging)

Recursion!

```
fac = lambda n: 1 if n == 0 else n * fac(n-1)

F = lambda fac: lambda n: 1 if n == 0 else n * fac(n-1)

F(fac) = fac   <- fac is the fixed point of F
```

$$F(YF) = YF$$

Fixed Points

$$F(YF) = YF$$

$$\omega = (\lambda x.xx)(\lambda x.xx)$$

$$\begin{aligned} \omega' &= (\lambda x.F(xx))(\lambda x.F(xx)) \\ &= F((\lambda x.F(xx))(\lambda x.F(xx))) = F(\omega') \end{aligned}$$

$$\boxed{Y = \lambda F.(\lambda x.F(xx))(\lambda x.F(xx))}$$

Puzzle

What is X? (There are 26 Ls)

Hint: A fixed point combinator has the form YF=F(YF)

$$X = (LLLLLLLLLLLLLLLLLLLLLLLLLL)$$
$$L = \lambda abcdefghijklmnopqstuvwxyzr.r(thisisafixedpointcombinator)$$

$$X = \lambda r.r(Xr)$$
$$XF = F(XF)$$

Conclusion + Acknowledgements

Everything + more:
http://xuanji.appspot.com/isicp/lambda.html
Lambda interpreter:
https://github.com/tarao/LambdaJS